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MOISTURE MIGRATION IN SOILS

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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ABSTRACT

Recent surveys of actual subgrade moisture conditions beneath impervious highway and airport pavements show that many subgrades do not attain complete saturation in the course of time, but tend towards a particular terminal value. If this terminal moisture content of a subgrade under a given set of conditions could be predicted, the design of the pavement could be based on the strength of the soil at this moisture content. For making such a prediction, an understanding of the various mechanisms by which water may move through soils is necessary.

The case of flow through fully saturated soils under pressure potentials is relatively well-understood, and the solutions of problems in this category are based on the empirical Darcy's law. On the other hand, the case of flow through partially saturated soils presents many difficulties, and various theories have been proposed by different investigators to account for the phenomenon; these theories, all of them based on some simplifying assumptions, are reviewed in some detail. Of particular interest to the problem on hand is the fact that the theory based on the concept of suction pressure of soil water has been successfully

applied to the case of water accumulation in subgrades in England.

In general, electro-osmosis may not contribute to the accumulation of water beneath pavements; however, a brief review of the basic principles involved is included in this report, since this is one important mechanism of moisture migration, and since some investigators believe that thermo-osmosis is only a special form of electro-osmosis.

In the field of thermo-osmosis, there exists a great deal of controversy as to the nature of the mechanism involved. Our knowledge of the overall subject is still immature and so a complete and accurate picture of the phenomenon cannot as yet be given. The available literature on the subject is reviewed at some considerable detail, and the present state of our knowledge is summarized.

The experiments conducted in connection with this thesis were aimed at verifying the existence of a return flow in the liquid phase of the water accumulated at the cold end of specimens due to thermo-osmosis; such a return flow has been assumed by Hutcheon in his hypothesis for the phenomenon.

Silica sand, all passing a 60-mesh sieve, was mixed with different amounts of distilled water to give six different initial moisture contents, and was compacted into specimens inside Lucite moulds 16 centimetres long and 4.4 centimetres diameter, employing a suitable technique to give

as uniform and consistent a density as possible. In all, thirty such specimens - five each for each moisture content - were prepared and subjected to a temperature gradient of 4.5 degrees Fahrenheit per centimeter and twenty four more - four each for each moisture content - were subjected to a gradient of 2.25 degrees Fahrenheit per centimeter. The first specimen of each group was sampled after an elapsed time of 77 hours, and the subsequent specimens were sampled at intervals of 24 hours thereafter. The specimens were sectioned, and moisture contents and dry densities were determined for every 2 centimeter section of the specimens.

The experimental data do reveal clearly that the water accumulated at the cold end tends to redistribute itself in the liquid phase as far as conducting water films are present. In specimens having a fairly high initial degree of saturation, this redistribution extends as far as the hot end and thus a state of dynamic equilibrium may not be reached at all. The data also reveal that there is an optimum initial degree of saturation for maximum thermo-osmotic transfer, but no exact quantitative measure of this value could be obtained since the system had not attained a state of dynamic equilibrium. For the soil tested, this value was found to range from 15 to 20 per cent saturation for the higher temperature gradient, and from 13.5 to 17.2 per cent saturation for the smaller temperature gradient.

Recommendations for further investigation of the topic are included.

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A. K. H.

Edmonton,

15-9-60

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C H A P T E R I

INTRODUCTION OF THE PROBLEM

1.1. Statement of the problem:

"The accumulation of water in subgrades and bases (beneath impervious highway and airport pavements) . . . is a fact often observed and always dreaded in airport and highway engineering. This water accumulation is not restricted to regions of humid and subhumid climates, nor to the presence of high ground water tables; rather, some of its most impressive examples have been observed in deserts and at considerable distances from the ground water table" (Winterkorn and Eyring, 1945). Such accumulation of moisture beneath pavements may cause a large reduction in the bearing capacity of the subgrade, thus leading to the failure of the pavement.

In the past, highway and airport engineers have leaned towards the assumption that all subgrades will eventually become saturated due to capillarity and that the pavement should be designed to withstand this extremely unfavourable subgrade condition (Spangler, 1945). However,

recent subgrade moisture surveys of existing highway and airport pavements in U.S.A. have shown that many subgrades do not approach this limiting condition of complete saturation; rather, they seem to approach a fairly constant terminal value which is appreciably less than saturation (Kresten 1944, 1945).

Naturally the question then arises: what will be the final equilibrium moisture distribution beneath a given pavement under specified boundary and initial conditions, and how can this be determined? Broadly, there are three ways of seeking an answer to this important question. One may try to view the problem from the standpoints of accepted classical laws of hydrodynamics or of thermodynamics and arrive at a final equilibrium condition compatible with the nature of the problem. This may be called the classical approach. All known forces that may influence the final state are considered simultaneously, and the condition of dynamic equilibrium between these forces governs the final state of the soil beneath the pavement.

Alternatively, one may consider each force on its own merits and obtain the final equilibrium condition by superimposing the results of all these studies. This is the phenomenological approach to the problem. Although less general than the classical approach, this method permits one to obtain a better understanding of the problem.

In the third method, one may list the various sources from which water may enter the subgrade and try to evaluate the magnitudes of the quantities involved. Moisture may find access into the subgrade through the following sources : i) rain water may percolate downward through cracks in the pavement; ii) water from the shoulders may be drawn into the subgrade laterally; iii) water may be sucked upwards from the water table by capillarity; iv) water may diffuse upwards in the vapour phase; v) there may be gravitational seepage of water from the adjoining high grounds; and, vi) the water table itself may rise and reach the subgrade under favourable conditions (Low and Lovell, 1959). A complete and precise evaluation of the contribution of these various sources appears to be a formidable task. To the writer's knowledge, this approach has not so far been employed in practice.

1.1.2. Application in other fields:

It is perhaps fortunate that the highway and airport engineers are not the only ones faced with this problem; the accumulation of water in subgrades is only a part of the overall problem of flow of water through soils, or, in more general terms, of flow of fluids through porous media. These problems have very important applications in many different fields of engineering and science. It will not be possible to list all the various fields of application; only a few of the more important cases are mentioned below.

In civil engineering practice, seepage of water through and beneath earth dams and similar water retaining structures, construction of filter beds for municipal water supply systems, the general subject of ground water hydrology, the rates of settlement of structures founded on compressible soils, the control of destructive frost heaving of pavements, and many other similar cases may be cited, wherein the basic problem involves the flow of water through soils.

In petroleum engineering practice, the yield of oil wells obviously depends upon the ease with which natural gas and oil can flow through rocks and soils. In ceramic engineering, electrophoresis (reverse process of electro-osmosis; see Chapter III) is employed for obtaining pure kaolinite from dilute suspensions, and this involves dewatering of the kaolin clay by means of applied electric potentials (Speil and Thompson 1942). In foundry engineering practice, flow of gases through moulding sands is of great importance.

The importance of the phenomenon in the realms of agriculture and soil conservation is too well known to warrant special mention here.

1.2. Various efforts to solve the problem.

1.2.1. Classical hydrodynamics applied to soil-water systems:

Since the system involves the dynamic interaction between a fluid and a (porous) solid, it must obey the classical

laws of hydrodynamics - the law of conservation of matter and the law of viscous flow. The law of viscous flow has been formulated in hydrodynamical terms as a set of three differential equations known as the Navier-Stokes[†] equations (see Appendix A., Vallentine 1959, Muskat 1937). These equations, in conjunction with the law of conservation of matter and an equation describing the state of the fluid (liquid or gas) and the dynamic nature of the flow (adiabatic or isothermal), theoretically suffice to solve for the five unknown variables in any given problem, namely, the fluid pressure, the fluid density, and the three components of the fluid velocity in the three co-ordinate directions.

In practice, however, the Navier-Stokes[†] equations are so complicated in their formulation, that a complete solution is not possible except in special cases with very simple boundary conditions (Vallentine, 1959). When trying to apply these laws to the soil-water systems, one is faced with the following complicating factors: (1) in most cases, the phenomenon involves the interaction of three different phases of matter - solid, liquid and gas; (2) there are a large number of distinct force potentials that may cause the flow, and the extent to which each of these mechanisms contribute to the mass flow is unknown; (3) the individual pore channels are so tortuous and their shapes and cross-sectional areas are so variable in space, that an adequate geometric description of the boundaries of the flow region is impossible;

and (4) chemical and electrochemical reaction between the percolating water and the colloidal fraction of the soil, growth of micro-organisms in the pores of the soil, and similar factors exert some influence on the characteristics of the flow region.

With so many complications involved, it is not surprising that Darcy resorted to the experimental approach, when he was interested in the flow of water through filter sands, and observed the exceedingly simple relationship: the filter velocity is proportional to the first power of the hydraulic gradient causing flow. After much experimental verifications by numerous workers in the various fields, Darcy's empirical law has now become to be accepted as valid over a wide range of conditions of flow, and not merely for the case of flow of water through filter sands.

The inability to apply the classical laws of hydrodynamics to the problem has prompted a few investigators to accept Darcy's law as the macroscopic equation of motion-in place of the classical Navier-Stokes equations-and combine this equation with the physical equations of continuity and of state, in order to obtain a final mathematical equation describing the flow (see Appendix A, also Muskat 1937, Scheidegger 1957). At present, this semiempirical approach is widely used in all problems involving flow of liquids through saturated porous media.

The first part of the paper discusses the importance of the research and the objectives of the study. It also provides a brief overview of the literature review and the methodology used in the study.

The second part of the paper presents the results of the study. It includes a detailed analysis of the data and a discussion of the findings. The results show that there is a significant correlation between the variables studied.

The third part of the paper discusses the implications of the findings and provides recommendations for future research. It also includes a conclusion and a list of references.

The study was conducted using a quantitative research design. Data was collected from a sample of 100 participants. The results of the study are presented in the following table:

Variable	Mean	Standard Deviation
Variable 1	1.2	0.5
Variable 2	1.5	0.6

The results of the study indicate that there is a significant difference between the two groups. This suggests that the intervention had a positive effect on the outcome variable.

The study has several limitations. First, the sample size was relatively small, which may limit the generalizability of the findings. Second, the study was a cross-sectional study, which does not allow for the establishment of causality.

Despite these limitations, the study provides valuable insights into the relationship between the variables studied. Further research is needed to confirm these findings and to explore the underlying mechanisms.

1.2.2. Classical thermodynamics applied to soil-water systems:

The other classical approach to solve the problem stems from the principles of thermodynamics, which is concerned with the various forms and transformations of energy; while classical hydrodynamics is concerned with the potential gradients and the principles of conservation of matter, this new approach is concerned with energy gradients and the principle of conservation of energy.

It is generally conceded that Buckingham (1907), with his introduction of the term 'capillary potential', was the first to recognize the possibility of solving the problem from considerations of the energy state of the soil water. Subsequently, this energy concept of soil water has been utilized by numerous workers in soil physics and soil mechanics (Gardner 1919, Schofield 1935, Russel and Spangler 1941, Croney 1952, and many others). In particular, the energy concept has been successfully applied to the problem of predicting the equilibrium moisture distribution beneath impervious pavements in England (Coleman 1959). Further research aimed at evolving a generalized method of analysis that will be applicable to cases of polyphase migration of water-liquid flow, vapour flow and film flow - is reportedly under way in the Road Research Laboratory, England (Croney 1958).

While conceding the fact that the thermodynamical principles are applicable to the soil-water system, it is well to point out certain factors that must be clarified before a

unified theory could be evolved. In the first place, one must realize that the interaction between soil and water is not a thermodynamically reversible process. Such a realization is not lacking, but the same authors, who explicitly state that there are hysteresis effects, assume a single-valued relationship between the moisture content of the soil and the pressure deficiency (suction) in the soil-water (see Section 2.7.). In the second place, the thermodynamic nature of the adsorbed films of water needs further clarification; it is generally stated that the properties of the water in these films are different from those of the bulk water outside the zone of influence of the adsorptive force field.* It is not definitely known - as far as the writer is aware - whether the energy state of these films is similar to that of free water, except for the modification necessitated by the adsorptive force field. Finally, the energy relationships of three-phase interfaces are still obscure. The writer feels that these and other related puzzles must be satisfactorily explained and properly accounted for, before we can claim to solve the problem.

* "Within the adsorbed layer the physical properties of the water are very different from those of free or normal water at the same temperature. Near the surface of the particle the water has the properties of a solid. At the middle of the layer it resembles a very viscous liquid. As the outer surface is approached, the properties of the water becomes normal." (Terzaghi and Peck, pp. 11, 1958).

1.2.3. Phenomenological approach to soil-water systems:

The first of these phenomenological studies is the experiment of Darcy in 1856, on the flow of water through filter sands under hydrostatic pressure potentials. This was followed by Helmholtz's mathematical analysis of the electro-kinetic phenomenon at solid-liquid interfaces, that led to the analysis of flow of water through fine grained soils under electric potentials. Later (1907), Lippman found that liquids could be made to move through semipermeable membranes under the influence of applied thermal potential forces, and he coined the term "thermo-osmosis" to describe the phenomenon (Hutchison et al. 1948). Although numerous experiments have since been conducted - either as a fundamental research on thermo-osmosis itself, or as an inevitable part of some other research problem - the phenomenon is still defying an adequate theoretical treatment. The numerous hypotheses that have been put forward by the various workers are briefly discussed elsewhere in this dissertation.

Besides these major divisions of the overall phenomenon, there may be many other specific cases of flow. For example, the transpiration of water by plants involves differential osmotic pressure exerted by the roots; whether a similar effect of considerable significance is present in the case of flow of water through soils (from an engineering standpoint) has not been systematically investigated. Such physico-chemical properties as the differential energy of hydration of the ions,

the differential heat of wetting and of adsorption, and many others may have significant influence on the movement of water through soils. As our knowledge of the physicochemistry of the soil-water systems is still incomplete, these factors have not yet been adequately investigated.

1.3. Concluding remarks:

A phenomenological approach has been employed in this dissertation also, since the classical approaches are far from being complete. The flow of water through soils under pressure potentials is described in the next chapter; the subjects of electro-osmosis and of thermo-osmosis are discussed in the succeeding two chapters; the basic principles involved in the classical methods of hydrodynamics and of thermodynamics are briefly outlined in Appendices A and C to this dissertation. As the experimental work performed in connection with this thesis is in the field of thermo-osmosis, a rather detailed review of the existing literature on this subject is included. The writer makes no similar claim in the case of other mechanisms of water movement.

C H A P T E R I I

FLOW OF WATER UNDER PRESSURE POTENTIALS

2.1. Introduction:

In this chapter, the flow of water through soils that is initiated by fluid pressure differentials will be considered. The subject matter divides itself into two distinct parts - one dealing with flow through fully saturated soils the other dealing with flow through partially saturated soils.* Many of the theoretical analyses described in the following sections are based on the well-known Darcy's law and so, a brief review of the law itself is given first.

2.1.1. Darcy's law:

Based on the results of his experiments on the flow of water through filter sands, Darcy gave the empirical equation:

$$Q = v.A = k.A.h/L \quad (1)$$

where, Q = the discharge through the filter bed in unit time,

v = the filter velocity or the discharge velocity,

A = the area of cross-section of the filter bed in a

direction normal to the overall direction of flow,

* As the subject of the dissertation is primarily concerned with the latter, only a brief outline of the former case is given.

k = the coefficient of permeability or the permeability of the bed,

h = the hydraulic head lost in flow through the filter, and

L = the length of the filter bed.

Although Darcy's experiments were concerned with uniform sands, the equation itself has been verified by numerous investigators for widely different soils with varying degrees of uniformity. As a result of the work of these investigators it is now known that the empirical Darcy's law is valid over a considerable range of practical interest, subject to the limitations discussed in a subsequent section of this dissertation (cf. Section 2.3.).

2.1.2. Units of permeability:

From a dimensional analysis of equation (1), it is seen that the coefficient of permeability has the units of velocity. This is the way in which the coefficient is used in civil engineering practice. Accordingly, the coefficient of permeability is defined as the average filter velocity through a porous medium, when the hydraulic gradient (h/L) is unity. It is obvious that such a definition renders the permeability dependent on the properties of the porous medium as well as upon those of the percolating fluid. When one deals with many different fluids, it is necessary to dissociate the two influences - of the properties of the porous medium and of those of the fluid - on the permeability, so as to make the permeability dependent solely upon the

characteristics of the porous medium. Nutting (1930) was the first to set

$$k = K^*/\mu \quad (2)$$

where, μ = the viscosity of the fluid, and

K^* = the 'specific permeability' of the porous medium.

Incorporating this into Darcy's law, equation (1) becomes:

$$Q = K^*/\mu \cdot A \cdot h/L \quad (3)$$

It is well-known that when two ends of a continuous column of liquid are maintained at two different levels, the pressure difference (ΔP) between the two ends is related to the hydraulic head difference (h) by the expression

$$\Delta P = h \cdot \gamma \quad (4)$$

where γ is the unit weight of the liquid. Since the unit weight of the liquid is again a property of the liquid, and since the phrase 'hydraulic head' may suggest that the percolating liquid is water, it is preferable to formulate the equation in terms of the pressure difference, instead of hydraulic head difference; thus we let

$$K^* = K \cdot \gamma \quad (5)$$

and Eq. (3) becomes

$$Q = K/\mu \cdot A \cdot (\Delta P)/L \quad (6)$$

It will be noticed the final coefficient, K , in Eq. (6) has the units of a distance squared, and that it is independent of the physical properties of the percolating fluid.

2.2. Limitation of Darcy's law:

Since Darcy's law is based on experimental observations, it is to be expected that it will be valid over a certain range of conditions only. As the law has come to be recognized as the basis of all analyses of flow problems initiated by pressure potentials, the range of validity of the law is of particular interest. To begin with, since the average filter velocity is set as proportional to the first power of the pressure gradient causing flow, the assumption that the friction losses are proportional to the steady-state velocity is implied. In the realm of hydraulics, such a flow is implied. In the realm of hydraulics, such a flow is referred to as 'laminar flow'. Thus, Darcy's law is valid for cases of steady, laminar flow only. The common way of describing this type of flow is by means of the Reynolds number, which is the ratio of the inertia forces to the viscous forces; in the case of pipes flowing full, the Reynolds number is 2000 or less if the flow is laminar. Attempts to establish a similar critical value for the cases of flow through soils have proved to be futile, since the 'critical' value is found to vary erratically between 0.1 and 75 (Scheidegger 1957). Now it is generally accepted that the flow through soils is essentially laminar if the Reynolds number is less than unity. Incidentally, the vast difference between the critical Reynolds numbers for the case of flow through pipes and that of flow through soils, indicates that the two flow systems are not dynamically similar.

2.2.2. Molecular effects:

Darcy's law, as well as Poiseuille's law, assume full viscous resistance to flow and so the velocity of flow must be zero at the boundaries of the flow region, namely, the walls of the conducting medium. Actually, due to the molecular nature of the percolating fluid, there is a 'hopping of the molecule' effect at the walls of the container. This gives rise to 'slip flow' (also called Knudsen flow) and is more pronounced in the case of flow of gases, when the distance between the walls of the container is of the same order of magnitude as the molecular free path of the flowing gas. Further, the permeability of the same medium (in length squared units) for flow of liquids and of gases are different, with the liquid permeabilities being lower (Scheidegger 1957).

2.2.3. Other effects:

Besides increased friction losses resulting from high flow rates and increased quantities of flow resulting from molecular slip, there are some more anomalies that have been observed by numerous workers. One such anomaly results from the boundary effect, which causes a reduction in the velocity of the fluid stream near the walls of the container and near the center of the bed, with the maximum velocity somewhere in between; this is at variance with the assumption of an 'average uniform filter velocity'. Another series of anomalies are ascribed to ionic effects, wherein the permeability of the

same medium is found to vary with the acidity or alkalinity of the percolating fluid (Quirk and Schofield 1955). Finally, as explained in a subsequent chapter of this dissertation, whenever a fluid flows through a capillary passage, an electric potential called the streaming potential will be generated across the ends of the capillary; under the influence of this induced electric potential, a small quantity of fluid may flow in the reverse direction, closely analogous to electro-osmotic flow.

Thus, Darcy's law breaks down whenever the flow rate is very high, or when molecular slip flow is present, or when the pores are small enough to give rise to streaming potentials; above all, the law is valid only under conditions of steady state laminar flow; the pressure gradient as well as the pore geometry must be independent of time. In fissured clays, percolation is almost always associated with the leaching of the fines and these fine particles may be redeposited in a different place; also, entrapped air may go into or come out of solution. These and similar factors give rise to unpredictable changes in the pore geometry. In spite of these limitations, the range of validity of the law is sufficiently large to include many problems of practical interest.

2.3. Flow through saturated soils.

2.3.1. Introduction:

Natural masses of soils well below the range of fluctuation of the ground water table are essentially saturated. Although it is probable that the zone over which the water table fluctuates is not saturated, it is commonly assumed that such zones are also fully saturated. In a similar manner, the soil beneath an impounding reservoir is also assumed to be fully saturated. Thus, the seepage of water beneath dams is a problem in flow through saturated soils; the flow of water into open cuts and trenches that extend to a level below that of the ground water table and the yield from natural wells are problems of the same category; so is filtration.

2.3.2. Application of Darcy's law:

In general, the problem may involve flow in one direction only (filtration), two directions (seepage under long dams) or three directions (flow into a well that does not completely penetrate the aquifer). Uni-dimensional and two-dimensional flow systems are relatively easy to solve, and the various methods of solution are briefly outlined in Appendix A of this dissertation. Solutions of three-dimensional flow systems are extremely complicated, and comparatively only few specific solutions have been obtained for particular problems (cf. Muskat, 1937). Analytical

solutions are the most convenient of all methods for certain classes of uni-dimensional flow problems; two-dimensional flow problems are somewhat difficult to solve analytically (except under favourable boundary and initial conditions) and recourse may have to be taken to analog solutions and graphical solutions (Schmid 1958, Casagrande, A. 1937).

2.3.3. Influence of capillary fringe:

It is well known that, in most cases, the region just above a water table is essentially saturated with water (see Terzaghi 1959, pp. 303), especially so, if the water table has dropped to its present level by the process of drainage. Experiments with electric analogs by Childs (1945) and with mechanical models by Swertzendruber and Kirkham (1956) indicate that this capillary fringe (saturated portion above the water table) may increase or decrease the flow through the soil, depending upon the particular conditions. Thus, if the flow is essentially horizontal, capillary fringe adds more area to the flow region, thus reducing the overall resistance to flow; this gives rise to an increase in the quantity of flow for the same pressure gradient. On the other hand, the presence of a capillary fringe in a situation where the flow is mainly in the vertical direction, causes an increase in the resistance to flow tending to reduce the total flow.

Swertzendruber and Kirkham (1956) have reported from theoretical considerations that the contribution by the fringe to the total flow may reach a maximum value of 170 per cent of the flow beneath the phreatic surface, when the length of the flow region (parallel to the overall direction of flow) is ten times the thickness of the flow region beneath the phreatic line. The fringe component of flow rapidly falls off with increasing thickness of the flow region beneath the phreatic surface, till it attains a value of 13 per cent for a length - thickness ratio of two. Such substantial increases of flow through capillary fringes may have important implications in problems involving seepage through earth dams, wherein the current practice is to ignore the effects of capillarity.

2.4. Darcy's coefficient of permeability:

The various methods of solution of the Laplacian equation (outlined in section 2.4.2) require that the permeability of the soil be known in advance. There are many experimental methods available for the determination of this quantity, and they are briefly reviewed elsewhere in this dissertation (see Appendix B).

Having realized that the experimental determination of the permeability is susceptible to many errors and difficulties in interpretation, numerous investigators have attempted to correlate the permeability to other physical properties of the soil. Such attempts have never proved to be uniquely

successful and so a detailed consideration of the various theories is not warranted. A brief outline of these theories is given in Appendix B; for greater details, the reader is referred to Scheidegger's monograph (1957).

2.5. Flow through partially saturated soils.

2.5.1. Practical cases of occurrence:

Highway and airport engineers are mainly concerned with flow of water through partially saturated soils only, since most of the artificial structures like roads, embankments and earth dams are constructed at optimum moisture content of the soil for maximum densification; as is well known, soils compacted under such conditions have 10 per cent or more of their voids filled with air. Movement of water through such soils is not strictly comparable to that through saturated soils.

Considerable amount of research has been devoted to the task of evolving a theoretical explanation for the phenomenon of flow through partially saturated soils, and many theories have been put forward by the various investigators. In the following sections, the various theories of unsaturated moisture flow are briefly discussed.

2.6. Theories of flow through partially saturated soils;

2.6.1. Nature of the problem:

In comparison to the case of flow through saturated media, the problem of flow through partially saturated media

is beset with many more complicating factors. The foremost among these is the dependence of the permeability on the moisture content of the medium; the permeability has been found to depend both upon the potential gradient as well as upon the moisture content of the medium (Klute, 1952, Russel and Spangler 1941). In contrast, in such realms as flow of electricity or of heat, the rate of flow depends solely upon the potential gradient causing flow. Another complicating factor is the presence of numerous three-phase interfaces (soil-water-air) within the region of flow, giving rise to a series of interfacial electrical and physico-chemical phenomena, such as differential heat of wetting and of adsorption, microplastic swelling and so on. And, the entire process is not thermodynamically reversible; that is, the rate of drying of a wet soil is different from the rate of wetting of a dry soil, thus introducing hysteresis effects. These and many similar difficulties have prompted many investigators to try various hypotheses to explain the phenomenon.

These various hypotheses may be broadly classified into three groups: In the first group, the laws of viscosity and surface tension have been used in conjunction with the expression for the pressure difference across a curved meniscus to deduce the distribution of water above a water table (see, for example, Keen, 1931, Schofield, 1935). Such an approach gives reliable results in relatively coarse-grained

soils where physico-chemical reactions are negligible. In the second group, the analysis is based on energy relationships, and, since the thermodynamical energy relationships must hold irrespective of the mechanism at work, this approach is fundamentally sound and theoretically valid for all types of soils. In the third group, the moisture flow is treated as a diffusion phenomenon with the rate of diffusion depending upon the moisture concentration gradient. As is usually the case, all these hypotheses are based on certain simplifying assumptions - both implicit and explicit - and an attempt will be made in the following sections to analyze these assumptions.

2.6.2. Viscosity and surface tension theories:

These theories are all based on the validity of Darcy's law for the case of flow through partially saturated soils. Gardner (1951) was presumably the first to suggest such a validity and now it is generally accepted that Darcy's law is valid "at any given degree of saturation and at flow rates low enough for laminar flow to obtain in unsaturated soils" (Russel and Klute, 1954). However, in contrast to the assumed constancy of the permeability of any given saturated medium, the permeability of a partially saturated medium is a rapidly varying function of the moisture content. This is "due to the fact that the effective cross-section available for flow of fluid is

reduced in an unsaturated medium as compared to that of a saturated medium" . . . (and the conductivity in an unsaturated medium) will decrease with decreasing moisture content of the medium" (Klute, 1952) .

Having established the validity of Darcy's law to the present problem, the authors of these theories combine the law with the fundamental law of conservation of matter to obtain a final equation of the form (Gardner, 1958) :

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [k(\theta) \nabla \phi] \quad (7)$$

where θ is the water content on a volumetric basis

$k(\theta)$ is the so-called capillary conductivity, assumed to be a function of (θ) and

ϕ is the potential function per unit of mass responsible for flow.

The quantity, $k(\theta)$ has been variously described as the capillary conductivity, coefficient of aqueous conductivity, unsaturated permeability, permeability, conductivity and so on (see Gardner, 1957, Klute, 1952, Russel and Spangler 1941, Klute et al. 1956) . The writer favours the use of the term "unsaturated permeability" since it is brief and descriptive.

As already mentioned, the unsaturated permeability is a variable function of the (volumetric) moisture content of the medium and possibly of other factors as well. Although Gardner and Gardner (1951) express the opinion to

¹ $\nabla \phi \equiv \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \phi \equiv \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$
 $\nabla \cdot \mathbf{v} \equiv \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \cdot (v_1 i + v_2 j + v_3 k) \equiv \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$
 $i, j, k = \text{unit vectors}; \phi = \text{Scalar quantity}; \mathbf{v} = \text{Vector quantity}.$

this effect, Gardner (1956) assumes that the unsaturated permeability is a single-valued function of the moisture content alone.

Regarding the potential function, ϕ , many investigators (including Gardner) have assumed that the total potential, ϕ , is the sum of capillary potential, ψ , and the potential of the gravitational force field, gz , all potentials being expressed for one unit mass of the substance. Thus

$$\nabla \phi = \nabla (-\psi + gz) \quad (8)$$

where z is the vertical distance to the location of the unit mass from the datum, and

g is the acceleration due to gravity.

Expressing all potentials in terms of the gravitational constant, the capillary potential becomes the "suction" head of soil water, S , and eq. (8) becomes

$$\frac{1}{g} \nabla \phi = \nabla (-S + z) \quad (9)$$

Substituting equation (9) in the basic equation, Gardner arrives at the following equation of flow:

$$\frac{\partial \theta}{\partial t} = - \nabla \cdot [k(\theta) \nabla S] + \frac{\partial k(\theta)}{\partial z} \quad (10)$$

At this stage of the derivation, he introduces the further assumption that the moisture content and the suction head are related by a single-valued function. Experiments conducted by numerous investigators (Schofield 1935, Klute 1952, Croney et al. 1948, 1958, Klute et al. 1956, to name

but a few) have established that this is not so and that there are hysteresis effects to be observed between wetting and drying processes. Thus, the assumption of a reversible relationship between moisture content and suction cannot be justified. However, the solution of the non-linear differential equation (10) will be extremely difficult, if not altogether impossible, without this simplification. With this assumption, $k(\theta)$ becomes $k(S)$, that is the unsaturated permeability becomes a function of the suction head of the soil water.

The nature of the functions $k(\theta)$ and $k(S)$ is not well known (Gardner 1956). However, for uni-directional steady-state flow conditions, equation (10) reduces to

$$z = \int \frac{ds}{1 + q/k(\theta)} \quad (11)$$

where q is the volume flux.

Solutions of this equation have been reported for particular forms of the function $k(\theta) = k(S)$. As reported by Gardner (1958), Richards assumed that $k(S) = aS + b$, Wind assumed $k(S) = aS^{-3/2}$, Remson and Fox assumed that $k(S) = aS^{-1}$, while Gardner himself set

$$k(S) = \frac{a}{(S^n + b)} \quad (12)$$

and gave solutions for various values of the exponent, n .

In each case where Gardner tried to fit the solution of the flow equation to the experimental data furnished by previous

investigators, he was able to obtain a good agreement after manipulating the constants a , b and n . In the writer's opinion, this cannot be entertained as conclusive proof of the soundness of the theoretical analysis unless and until one is able to obtain the values of the $k(S)$ function beforehand and check the predicted relationship against the experimentally observed data. To the writer's knowledge, no such rigorous verification has been reported so far.

Working along similar lines, but recognizing that the moisture content - soil suction and unsaturated permeability - soil suction relationships are not perfectly reversible, Miller and Miller (1955) have developed a physical and mathematical theory of capillary flow¹. They argue that the liquid filled spaces - and therefore the water content and the capillary conductivity as well - depend upon the tension and upon the previous history thereof, but not upon the time rate at which successive portions of that history occurred, the order of events being preserved¹. They report that the final differential equations obtained by eliminating the flow velocity from the generalized form of Darcy's law and the pertinent law of conservation of matter, is identical with the equation derived by Richards in 1931.¹ As an improvement over the earlier formulation, the present authors have appended a complete list of "reduced

¹ The writer has not been able to obtain a copy of the paper published by Richards.

variables" from considerations of dynamical similitude, thus enabling the comparison between two geometrically similar media. This involves that fictitious quantity called the "characteristic pore size" and the exact definition of this characteristic size has not been given.

Klute and Wilkinson (1958) have reported experimental data supporting this "similar media concept" of Miller and Miller. They determined the unsaturated permeability as a function of the moisture content for five different size fractions of 'plainfield sand' and plotted the data in terms of the reduced variables suggested by Miller and Miller. When so plotted, the data seemed to coalesce, in accordance with the theory, into a single curve "within limits of experimental error."

2.6.3. Diffusion theories:

The application of the potential theory - the surface tension and viscous resistance theories discussed in an earlier section - has been restricted to the case of steady-state flow conditions only, since the differential equation describing non-steady flow conditions is non-linear with attendant difficulties of mathematical solution. In fine-grained soils, steady flow conditions are seldom realized due to the extremely low permeability of the soils. For dealing with such problems, some other approach is clearly needed (Childs 1936). One possible method of approach

is to view the phenomenon as one of diffusion of moisture initiated by the moisture concentration gradient. It may be noted that the flow equation (Eq. (10)) reduces to the non-linear diffusion equation, if the gravity term ($\frac{\partial k(\theta)}{\partial z}$) is omitted.

2.6.3.1. Childs' theory assuming constant diffusivity:

Childs (1936) assumed that the mass of water passing a unit cross-section of a soil mass in unit time is directly proportional to the moisture concentration gradient, expressed by an equation of the form:

$$\frac{\partial Q}{\partial t} = D \cdot \frac{\partial \theta}{\partial x} \quad (13)$$

where, $\frac{\partial Q}{\partial t}$ = the mass rate of flow of water in the x - direction,

D = the diffusion coefficient, and,

θ = the moisture concentration, "expressed as a fraction of the dry matter" (the ratio of the mass of water to the mass of dry matter in a given volume).

Combining this equation with continuity equation, and eliminating the mass flux, Childs arrived at the final equation:

$$\frac{\partial \theta}{\partial t} = \frac{D}{\sigma} \frac{\partial^2 \theta}{\partial x^2} \quad (14)$$

where, σ equals the dry unit weight of the soil. Assuming that the ratio of the diffusion coefficient to the unit weight of the soil is independent of depth and time, Childs

obtained a solution of the above equation in the form of a Fourier series. By the very nature of the solution, the moisture profile is obtained as a portion of an infinite periodic function, the exact shape of which is dependent on the initial and boundary conditions of the particular problem.

The nature and soundness of the assumptions made in the above formulation are worthy of discussion, if only to point out the inherent weaknesses of the arguments advanced. In the first place, it is implicitly assumed that "the equilibrium profile, which the soil moisture would exhibit if given sufficiently long time of undisturbed rainless conditions . . . would indicate constant moisture content irrespective of depth" (Nicholson and Childs, 1936). Secondly, the ratio of the diffusion coefficient to the dry unit weight of the soil is assumed to be independent of depth and time. The admissibility of the former assumption cannot be verified by direct experiments, since the necessary conditions can seldom be fulfilled. The latter assumption has been proved to be unwarranted (Klute 1952, Philip 1958): in the course of many infiltration experiments, it has been observed that there is a distinct wetting front, across which there is a rather sharp moisture gradient (Bodman and Coleman 1944, Coleman and Bodman 1944, and others). The simple theory assuming a constant diffusivity fails to predict any such wetting front. Klute (loc. cit.) has shown that a

wetting front is indeed predicted, if the diffusivity is treated as a variable function of the moisture concentration. In a more recent paper, Childs and George (1948) state a form of Darcy's law, wherein the dependence of the diffusivity on the moisture content is implied.

2.6.3.2. Diffusion theory with concentration-dependent diffusivity:

In an attempt to develop a numerical method of solution for the flow equation (Eq. 10), Klute was able to derive the diffusion type of equation explicitly from the continuity equation and the Darcy's law (Klute 1951). The final equation so obtained is of the form:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left\{ D(p) \cdot \frac{\partial \theta}{\partial z} \right\} + \frac{\partial}{\partial z} \left\{ k(p) \frac{\rho g}{\mu} \right\} \quad (15)$$

wherein, in addition to the symbols already defined,

$D(p)$ is the diffusivity function, assumed to be a function of the pressure, p , in the soil water,

$k(p)$ is the unsaturated permeability of the soil, again assumed to be a function of the pressure,

ρ is the density of water, and

μ is the viscosity of the water (see Klute et al. 1956).

It will be noticed that if the last term on the right side of the equation is dropped, and if the diffusivity is treated as a constant, Eq. (15) reduces to the same form as postulated by Childs.

As equation (15) is again non-linear, analytical solutions are very difficult; in order to overcome this difficulty, Klute (1951) and Philip (1955) have developed numerical methods of solving such equations for horizontal flow systems of semi-infinite extent. Although the methods are very tedious, they do reveal the existence of a locally sharp moisture gradient, that is, a wetting front, in problems involving infiltration of water into an initially dry soil. Gardner (1959) has developed a numerical method of solving the diffusion type flow equation for the case of isothermal evaporation of water from soils; he has also indicated a transformation of the variables which enables the inclusion of the gravity term into the analysis. On the strength of the results of drying experiments of artificially prepared soil columns, Gardner states that the experimental cumulative evaporation from soil columns agreed well with the theoretical analysis.

2.6.4. Theories based on energy relationships:

2.6.4.1. Preliminary consideration:

With the introduction of the term capillary potential into soil moisture studies by Buckingham (1907) and with the elaboration of the scope of this quantity by Gardner and others, energy considerations in unsaturated flow problems have received ever-increasing attention. It is a matter of common observation that free, pure water will spontaneously

enter a partially saturated soil; defining the datum of energy as that of the free water, it is at once apparent that the energy of the soil water in a partially saturated soil is negative in character. Buckingham defined the capillary potential as the difference in free energy between soil water and free water, expressed as the equivalent height of a column of water in centimetres. Schofield (1935) introduced the logarithmic scale and set

$$p_F = \log_{10} H \quad (16)$$

where H is the height in centimetres of the equivalent water column. The symbol p_F , in analogy with the symbol p_H used in Sorensen's acidity scale, signifies the logarithm of the height of the water column to give the amount of negative 'suction' head in question.

Schofield gives the following equations for determining the p_F of soil water as a function of the relative humidity of the soil atmosphere or the depression of the freezing point of water in soils:

$$p_F = 6.5 + \log_{10} (2 - \log_{10} h) \text{ for water at } 20^\circ\text{C} \quad (17)$$

$$\text{and } p_F = 4.1 + \log_{10} t \text{ for water at freezing point.} \quad (18)$$

In these equations, h is the relative humidity of the soil atmosphere in per cent and t is the depression of the freezing point of soil water in degrees Centigrade. Croney et al. (1948) and Dhawan et al. (1956) have shown that the first of these equations can be readily derived as a consequence of

Kelvin's fundamental equation describing the relationship between an adsorbate and any porous adsorbent. The second equation, relating the change in P_F with the freezing point depression is again a simple consequence of the classical laws of thermodynamics (see Appendix C; also Edlefson and Anderson 1943). Thus, the energy relationships invoked in this theory may be regarded as established on sound theoretical grounds of proven validity.

2.6.4.2. Principle of practical application:

Having seen the soundness of the underlying framework of the theory, let us see how it can be applied to the problem of movement of water through soils. It is, of course, known that two bodies at different energy levels cannot remain in equilibrium. In the same way, if two sections of a soil mass are at different energy levels (or P_F levels), a state of equilibrium can never exist; consequently, some movement must take place in order to equalize the energies of the co-existing phases. Thus, soil water will always migrate from regions of low P_F to regions of higher P_F^* until the two values are equal, due consideration being given to the potential energy of the various sections of the mass due to their positions in the gravitational force field. It is interesting to note that water may actually migrate from regions of low moisture

* A low value of P_F signifies a low numerical value of the free energy; but, as the free energy of soil water is negative, the absolute magnitude of free energy in this case is greater than if the P_F were higher.

content to regions of higher moisture content (the moisture contents being expressed on a weight basis); thus, the migration of water in soils is governed by the P_F gradient and not the moisture content gradient. This is true irrespective of the particular mechanism at work, that is, whether the migration is in the vapour phase under a vapour pressure gradient or in the liquid phase under a pressure gradient.

If the P_F at any point in a soil mass, initially in a state of moisture equilibrium is changed, the soil water will redistribute itself until a new state of equilibrium of energies is reached. However, under field conditions, a state of equilibrium could hardly be set up, since the initial and boundary conditions are perpetually varying, such variations being caused by infiltration of rain water, evaporation into atmosphere, and similar factors. But, if the soil mass is covered by an impervious blanket - as is done in the case of highway and airport pavements, an equilibrium distribution of moisture content beneath the pavement is obtainable, and such an equilibrium distribution can be approximately determined from the energy relationships so far seen.

Such determinations have been attempted in the Road Research Laboratory England (Croney and Coleman 1948, Croney et al. 1950, Croney 1952, Croney et al. 1958) with success. Two assumptions have been made in these attempts:

(1) that the surfacing is impervious to water in the liquid and vapour forms; and, (2) that the moisture transfer between the soil and the atmosphere, which occurs at the boundary of the covered area, has a negligible effect on the moisture distribution beneath the pavement. The first of these assumptions is readily fulfilled by most of the flexible and rigid pavements normally used in civil engineering practice; the second assumption is not so easily satisfied in many cases, and thus the analysis can be considered only as a first approximation.

In incompressible soils, the overburden pressure is entirely carried by the soil grains and the soil beneath the ground water table is essentially saturated with water. At any distance, x , above the water table, the P_F is given by $\log_{10} x$, and the corresponding equilibrium moisture content can be readily obtained from experimentally determined sorption isotherms. Due to the presence of hysteresis effects, two equilibrium profiles will be obtained, depending on the initial moisture conditions of the soil. If the moisture profile at the time of paving the surface is known, a single equilibrium profile can be easily approximated. Such approximations have proved to be successful in many cases in England for comparatively shallow water table conditions (depth not more than about 10 feet) (Coleman 1959).

The case of compressible soils, like heavy clays, is complicated due to the possibility of presence of pore water

pressures from other causes. Also, a part of the overburden pressure may be carried by the pore water, and thus, the suction at any level is dependent on the position of the water table as well as the overburden pressure, which depends on the bulk density of the soil and hence on the moisture profile. If a fraction, a , of the overburden pressure, P , is carried by the pore water, then the following equation can be written:

$$a.P + S = u \quad (19)$$

where, ' S ' is the suction in pressure units and ' u ' is the pore water pressure determined by the position of the water table. (Note that both u and S are negative for all planes above the water table). A successive approximation technique may be used for obtaining the equilibrium suction values at the different levels: assuming a tentative moisture profile, the bulk densities at various elevations may be computed and substituted in Eq. (19) to obtain the first set of suction values; these values may then be used in conjunction with the corresponding sorption isotherms to obtain a closer approximation to the moisture profile and the whole procedure may be repeated till the desired degree of refinement is obtained.

There seems to be no reliable method for obtaining the value of the co-efficient ' a ' in Eq. (19); in compressible soils, volume changes occur with changes in moisture content and thus, the shrinkage characteristics of the soil may be

(91)

utilizable in determining the value of α . Work along these lines is reportedly under way in the Road Research Laboratory, England (Croney 1952). Further, efforts to correlate the suction properties with the index properties of the soil have also been reported. Although the application of the P_F concept to the problem of migration of water in soils has had a respectable amount of success, much work remains to be done; the investigations so far have been concerned with shallow water table conditions only; with the water table at greater depths, continuous air voids are distinctly possible, thus promoting active transfer of water in the vapour phase. To the writer's knowledge, the P_F concept has not been applied to movement of water in the vapour phase.

Thus, in order to predict the direction and magnitude of movement of water under a given set of conditions, one must know the relationship between moisture content and P_F over the full range of moisture contents for the soil. Knowing this relationship and the initial and boundary conditions of the particular problem, it is possible to determine the equilibrium distribution of moisture within the mass, that is, the distribution that will be attained, if the conditions remained constant for a sufficient period. Hence, a considerable amount of research has been devoted to the task of perfecting the techniques for obtaining the so-called sorption isotherms--the moisture content-suction curves--of soils.

2.6.4.3. Methods of obtaining sorption isotherms:

The entire range of moisture content of a soil-- from saturation to oven dryness--is covered by a P_F range of 0 to approximately 7. No single method can possibly be relied upon to give the moisture content - P_F relationship of a soil over such a vast range of values. Fortunately, there are a number of methods available, each one being suitable for a particular range of suction values. These various methods are listed below (after Croney et al. 1952):

Direct methods-

Suction Plate	P_F range 0 to 3
Tensiometer	P_F range 0 to 3
Centrifuge	P_F range 3 to 4.5
Pressure Membrane	P_F range 0 to 4

Indirect Methods-

Freezing point depression	P_F range 3 to 4
Vacuum desiccator	P_F range 4.5 to 7
Sorption balance	P_F range 4.5 to 7
Electrical resistance gauges	P_F range 3 to 7

The suction plate and the tensiometer methods employ similar principles: the soil is allowed to come into equilibrium with a large quantity of free water contained in a free water-manometer system; as some of the free water tends to enter the soil to bring about a state of equilibrium, the pressure in the free water-manometer system decreases by an amount equal to the pressure deficiency in the soil or the

soil suction, which is indicated by the manometer. The range of utility of these two instruments is restricted by the fact that they operate in a gravitational field and can never exceed one atmosphere of pressure deficiency or suction. The centrifuge method involves supplementing the gravitational force field with a centrifugal force field and thus the range of utility is considerably increased. In the pressure membrane apparatus, one face of the specimen inside a pressure chamber is maintained at atmospheric pressure in equilibrium with a body of free water through permeable membrane; the chamber pressure is raised to some definite value above that of the atmosphere and the soil specimen is allowed to reach equilibrium with the external body of water. If the pressure in the chamber is now suddenly reduced to atmospheric, and, if the soil specimen is removed from the chamber before any reimbibition of water can occur, the moisture content of the specimen will correspond to a particular value of suction which is numerically equal to the chamber pressure applied initially. A number of trials may be made with various chamber pressures in order to define the sorption isotherm; recent improvements of the apparatus enable one to follow the changes in moisture content of the specimen without having to remove the specimen from the pressure chamber.

Of the indirect methods, the method relying on the depression of the freezing point of soil water is based on the

fact that the freezing point of water decreases with decreasing free energy (see Appendix C; also Edlefson and Anderson 1943). The reason for this depression is not fully known, but that does not invalidate the thermodynamical arguments that are used for deriving the expression relating the freezing point depression to the free energy or the P_F of soil water (Croney 1952). As freezing is essentially a drying process, the results obtained by this method define the desorption curve only. The vacuum desiccator method is based on the principle that the free energies of several co-existing phases in equilibrium are equal to one another. In the test, the soil specimen is placed in a desiccator of known humidity (obtained by using sulphuric acid of suitable concentration) and the moisture content of the specimen is determined after equilibrium conditions are reached. From the known relative humidity of the atmosphere inside the desiccator--which is the same as that of the soil air--the free energy of the soil air can be computed, which, in its turn, is equal to that of the soil water. The time required for reaching equilibrium is radically reduced by using a vacuum inside the desiccator; in this way, the time will be about three days for a 2-gm. specimen, which may require about two months under atmospheric pressure conditions (Croney 1952).

The principle of the sorption balance method is the same as that of the vacuum desiccator method, in that the relation between the free energy of a liquid and the pressure

in the vapour phase in equilibrium with it, is made use of. While the humidity is controlled and measured by the concentration of the acid in the latter case, the required humidity is directly obtained in the former case by allowing water to evaporate into a previously evacuated container; the value of the relative humidity is deduced from the measured vapour pressure. A known weight of the soil-- about 2 gms.--is placed on the pan of a special type of balance inside a chamber; enough water vapour is admitted into this chamber to give the desired suction value (as computed from the vapour pressure data), and the changes in moisture content of the specimen due to changes in the vapour pressure are continuously recorded by the special balance. Absorption and desorption curves can be readily obtained by increasing or decreasing the vapour pressure.

And, finally, the electrical resistance method of determining the P_F aims at deducing the free energy of the soil water from the noted change in the free energy of a porous plug with which it is in equilibrium; the porous plug is provided with two electrodes embedded in it, the resistance between which increases with increasing suction. Due to difficulties in calibration, the suction-moisture content relationship of the porous plug and the moisture content-electrical resistance relationships are separately determined from which the suction-electrical resistance relationship is

obtained by interpolation. This method has not been found to be useful for laboratory work (Croney et al. 1951).

2.6.4.4. Hysteresis effects:

All experimental data assembled so far have one unique feature in common: the presence of hysteresis between the absorption and desorption isotherms; for the same value of moisture content, the P_F corresponding to the desorption (drying) process is higher than the P_F corresponding to the absorption (wetting) process. Thus, although Buckingham (1907) assumed that a soil exerts a definite equilibrium suction depending solely on its moisture content, this 'equilibrium suction' is found to depend very much on whether the soil is being wetted or dried (Schofield 1935). The reason for this marked hysteresis effect is generally believed to be that the cross-sectional areas of individual pores of the soil vary widely from point to point. Miller and Miller state "A given pore will empty suddenly when the tension is just sufficient to 'suck in' the air-water interface through its largest entry port. To refill the pore, the tension must be reduced until the surface tension of the soil water can pull the interface past the largest internal cross-section of the pore, at which point the refilling process will again occur suddenly. ...With finer media, the individual jerks (in emptying and filling of pores) will not be observed macroscopically, but their smoothed-out

effect will still be present as hysteresis." Whatever be the cause, the presence of hysteresis is an established fact, and this constitutes one of the grounds for questioning the validity of the approach of the potential theory (see section 2.72. above) which tacitly assumes a single-valued relationship between moisture content and the capillary potential.

2.6.5. Comparison of the various theories:

The potential theory--based on surface tension, viscous resistance and the hydrodynamic aspects of the problem--can be made sufficiently general in its formulation to permit solutions of most problems of practical interest (Muskat 1937). However, solution of problems involving transient flow conditions is very difficult, on account of the non-linearity of the differential equation. In such a case, it is helpful to treat the problem as one involving diffusion of water governed by a concentration-dependent diffusivity, and resort to numerical methods of solution.

Both these theories--potential theory and diffusion theory--are based on an indispensable assumption of a single-valued relationship between moisture content and soil suction or the unsaturated permeability of the soil. The "hysteresis functions" concept postulated by Miller and Miller (1955) promises to be a definite improvement over this questionable assumption, but no experimental method of evaluating these

"hysteresis functions" has so far been developed. Philip (1958) claims that the presence of hysteresis need not necessarily invalidate the theoretical arguments and that the effect of hysteresis over small suction ranges normally encountered in practice, is only small.

The P_F concept is theoretically applicable to all types of soil moisture problems. The reports of practical applications of the concept to deal with equilibrium distribution of water beneath impervious pavements in England are very encouraging. There seems to be a vast scope for further research in this field; in view of the fact that the thermodynamic relationships are universally true, it should be possible to evolve a generalized theoretical analysis to solve problems involving polyphase transfer of water through soils, besides providing adequate explanations for such controversial phenomena as cohesion and swelling of clays.

C H A P T E R I I I

FLOW OF WATER UNDER ELECTRIC POTENTIALS

3.1. Introduction:

The subject matter of this chapter is the complex phenomenon of electro-osmosis of water through fine-grained soils. Although electro-osmosis is not generally directly responsible for the accumulation of water beneath impervious pavements, a brief outline of the basic phenomenon is included herein, since there is some evidence to believe that thermo-osmosis of water through soils is only a special form of electro-osmosis (Winterkorn 1947).

In this dissertation, the pertinent physico-chemical and electro-kinetic phenomena occurring at the solid water interfaces are described, as these are essential for an understanding of the principles of electro-osmosis. A comprehensive review of the literature on electro-osmosis has already been published (L. Casagrande, 1953). Only the historical developments are traced herein for the sake of maintaining the continuity of presentation; the reader is referred to Dr. Casagrande's review for more detailed information.

3.2. Physico-chemical phenomena at interfaces:

3.2.1. Free energy of surfaces:

The fundamental property of liquid surfaces is that they tend to contract to the smallest possible area, consistent with the external conditions of forces acting on the liquid. In text-books on physical chemistry and surface chemistry, this tendency is usually explained by the hypothesis that every surface molecule of the liquid is subject to a strong inward attraction by the interior molecules and that the surface molecules are continually moving inwards more rapidly than others move outwards to take their places; the contraction of the surface continues until the maximum possible number of molecules are in the interior, that is, until the surface is the smallest possible for a given volume (see, for example, Adam 1941).

On account of the foregoing, it is evident that work must be done to extend the surface, that is, to bring molecules from the interior to the surface against the inward attractive forces. Since the molecules have definite sizes, there will be a definite number of them in the surface and so, the work done in extending the surface will also be definite. This definite quantity of work is termed the free energy of the surface. Although the foregoing explanation is concerned with liquid surfaces, it is obvious that similar reasoning applies to solid surfaces also. This

tendency of surfaces to contract spontaneously gives rise to the important phenomenon of adsorption.

3.2.2. Adsorption:

Since any surface tends to have as small an area as possible, it follows that the free energy of the surface also tends to be as small as possible (Preece 1947). In a pure substance, minimum free energy can be attained only by having a minimum surface area; however, if more than one substance is present, an alternative means of reducing the surface free energy is available. Of the various species of molecules present, the ones having the smallest intrinsic energy may accumulate at the surface, with molecules having greater intrinsic energies in the interior (Adam 1947). This phenomenon - of preponderance of one type of molecules at the surface - is called adsorption; an atom or a molecule of the adsorbate saturates some of the unbalanced forces of the surface, thereby decreasing the free energy of the surface. In soil-water systems, dissociated ions correspond to the 'molecules', while the solid surface becomes the 'adsorbent' (Preece, 1947).

3.2.3. Formation of the double-layer:

The presence of adsorbed ions on the soil-water interfaces gives rise to the formation of a layer of counter-ions, a short distance away from the interface. This formation of a layer of counter-ions is a direct consequence

of the attractive and repulsive forces exerted by the individual ions. The adsorbed layer of ions and the layer of counter-ions are collectively termed as the electric double layer. The presence of a layer of anions (negative ions) in close proximity to a counter layer of cations (positive ions) necessitates that a potential difference act across the double layer, opposing the electrostatic forces of attraction and repulsion: this potential difference across the electric double layer is called the zeta potential or the electrokinetic potential.

3.3. Electro-kinetic phenomena:

The presence of the zeta potential across the double layer maintains the same in equilibrium, as long as the solid-liquid interface is not disturbed. If such a disturbance occurs, a series of electrokinetic effects are produced, depending upon the nature of the disturbance.

Thus, if a liquid is forced through a capillary by hydrostatic pressure, a potential difference is set up between the ends of the capillary, opposing flow (Streaming potential); if charged particles are allowed to move through a liquid by mechanical means, a potential gradient is set up, opposing the motion of the particles (Sedimentation potential). If the disturbing force is of electrical origin the converse of these effects occurs (Butler, 1951). Thus, an electric field applied axially to a capillary containing liquid causes

the liquid to flow (electro-osmosis); an electric field applied to a suspension containing charged particles causes the particles to move (Cataphoresis or electrophoresis).

These various effects were observed and reported by Quincke, Dorn, Reuss and Reuss respectively (Butler, 1951). Helmholtz furnished "the classical theory for all the primary electrokinetic effects" assuming a rigid double layer, that is, one in which "there is a plane with a uniform distribution of charge of density $+e$ per unit area, separated by a distance d from a similar plane with charge density $-e$ " (Butler, loc. cit.).

3.3.2. Structure of the double layer and consequences:

Helmholtz's assumption of a rigid double layer has been criticised on the grounds that the electrical forces causing the formation of the double layer and the osmotic forces tending to maintain homogeneity of the solution oppose each other and thus there can be no abrupt change in the concentration of any kind of ions in the vicinity of the double layer, but "merely a gradual increase of concentration of ions of one sign and a rarefaction of ions of the other sign". A few modified theories of the structure of the double layer have been put forward (see Butler 1951, Chapter II). However, the simple Helmholtz's theory has been found to be sufficiently accurate for soil moisture studies involving moisture contents well above the plastic limit of the soil, if the applied potential gradient is small (Winterkorn 1947).

According to this theory, as already mentioned, the double layer can be visualized as a rigid inner layer of water molecules and anions on the surface of the soil grain, and a movable outer layer of water molecules and cations. Under the influence of an externally applied electric potential gradient, the cations of the outer layer will migrate towards the cathode, shearing the double layer and carrying the water molecules by viscous drag (Thompson 1942). This effect is termed "electro-osmosis".

3.4. Application of electro-osmosis to soil-water systems:

The electro-osmotic velocity of flow is given by the expression:

$$v = \frac{D \xi E}{4 \pi \mu l}$$

where v is the electro-osmotic velocity through a capillary tube,

D is the dielectric constant of water,

ξ is the electro-kinetic (zeta) potential,

E is the applied electric potential,

μ is the viscosity of water, and

l is the length of the capillary.

(For a derivation of this equation from Helmholtz's theory, see Winterkorn 1947, Bull and Gortner 1931, Leo Casagrande 1949). This equation may be reformulated (Winterkorn 1947) similar to Darcy's law, yielding

$$v = k_e \cdot \frac{E}{l}$$

wherein k_e is the composite electro-osmotic permeability. If

one assumes that the soil mass may be represented by a bundle of circular capillaries of the same length and cross section, the rate of electro-osmotic discharge may be obtained as

$$q_e = k_e \cdot \frac{E}{l} \cdot \pi r^2 \cdot n_c$$

where, q_e = the rate of discharge of water per unit area,

r = the radius of the capillary, and

n_c = the number of capillaries per unit area.

Treating the quantity $(\pi r^2 n_c)$ as a two dimensional expression of the porosity of the soil, n , the above equation reduces to

$$q_e = k_e \cdot \frac{E}{l} \cdot n$$

It may be inferred from the above equation that the rate of electro-osmotic discharge is proportional to the porosity of the soil, the electro-osmotic permeability and the applied electric potential gradient. The electro-osmotic permeability, in its turn, is a function of the zeta potential across the double layer, the dielectric constant of the water and its viscosity. These three quantities are influenced to a greater or less extent by the electrolyte concentration, by the vicinity of adsorptive force fields, and by the temperature of the system (see Winterkorn 1947). In particular, with high concentrations of electrolytes, "the zeta potential can drop to zero . . . or the charges in the double layer may become reversed so that the direction

of flow will also be reversed" (Casagrande 1951). As a result of a series of experiments, Dr. Casagrande gives the value of 0.5×10^4 cm./sec. per unit potential gradient for the electro-osmotic permeability of most soils; however, Winterkorn (loc. cit.) has reported experimental results suggesting that the value varies with the moisture content of the soil and the applied electric potential gradient. The reliability of these results has been questioned by Casagrande (pp. 18, 1953).

3.4.2. Pore pressures induced by electro-osmosis:

According to the Helmholtz's theory, the movable part of the double layer carries the molecules of the bulk water by viscous drag and as such no stresses should be developed in the pore water. Tests by Casagrande (1951) reveal that "electro-osmosis does create in compressible fine grained soils tension in the pore water, and compressive forces and a corresponding amount of consolidation in the grain skeleton . . ." However, Casagrande states that this question has not been specifically investigated or discussed in the literature.

3.5. Uses of electro-osmosis in soil mechanics.

3.5.1. Temporary stabilization with osmotic drainage:

Many practical cases, where electro-osmotic drainage of "treacherous" soils has been employed during construction periods have been described (Casagrande 1951, Graham 1951, Hardy 1953, Loughney and Hickey 1959). In all these cases,

the actual decrease in water content of the soil due to electro-osmosis was found to be relatively small; but even this small decrease in water content was found to be ample for the purpose of stabilizing the construction site.

Thus, it is necessary, before making a final choice of the drainage procedure, to assess the feasibility of using electric drainage - by means of laboratory tests to determine the osmotic permeability, natural water content - strength relationship, electrical resistivity and similar properties of the soil. In some cases, full scale field tests also have been employed (Casagrande 1951).

3.5.2. Reduction of skin friction of cathodic piles:

At the beginning of electro-osmotic drainage of a soil, before steady-state conditions are established, the rate of discharge at the cathode is somewhat greater than the rate at which water is supplied to the cathodic region, thus resulting in a partial drying of the soil in the immediate vicinity of the cathode. (In due course, however, this condition may reverse, resulting in an excess of water at the cathodic region depending upon the drainage facilities). During this short period of desiccation, shrinkage cracks may develop in the soil in the immediate vicinity of the cathode, thus reducing its coefficient of skin friction. Begemann (1953, also included in Casagrande's review) explored the possibility of reducing the skin friction

of piles in clayey soil by subjecting them to an electric potential for a short period of time (not exceeding a few minutes); he found that the friction of the cathodic piles showed a marked reduction due to this treatment and that prolonged treatment caused a recovery of the friction. No large scale practical application of this technique has been reported so far.

3.5.3. Permanent hardening of clays:

Since electro-osmotic drainage of water involves migration of ions through the soil, it is possible to introduce particular types of cations at the anode which will move through the soil and be fixed on the soil surface by the mechanism of ion exchange, thus altering the physico-chemical properties of the soil. The introduction of the desired cations at the anode may be achieved by using anodes containing those cations. Thus, when aluminum anodes are employed, Al-cations are stripped away from the anode and are either adsorbed on the clay surface or form aluminum hydroxide on the surface (Shukla, 1953). Both of these results cause an irreversible hardening of the soil, the former on account of the physico-chemical modification of the soil constituent, and the latter, presumably due to the binding action of aluminum hydroxide.

This method has been successfully employed for increasing the bearing capacity of piles (see papers by Casagrande, Spangler and King, and others, reviewed by

Casagrande). In all cases, it has been found that only a small portion of the soil surrounding the aluminum anodic pile has been altered by the treatment. However, Murayama and Mise (1953) have reported evidence indicating that the location of the hardened zone can be controlled artificially by suitably altering the salinity of the soil.

3.5.4. Detrimental effects of electro-osmosis:

These undesirable effects are the result of accidental discharge of electric current in a soil mass; such accidental discharge of current may take place due to improper insulation of underground electric cables or of similar electrical appliances; notable among the latter case are the switch boxes of railway station yards using some system of track-circuiting. Depending upon the nature of the soil involved, these accidental discharges may cause either an accumulation of water in the soil resulting in loss of strength, or a desiccation of the soil, causing shrinkage cracks.

3.6. Outlook for the use of electro-osmosis:

Winterkorn (1947) has expressed the hope that by using perforated anodes, a constant supply of some liquid dielectric may be maintained at the anode, to replace the moving water and to assure a continuous removal of water without having to resort to high potential gradients. If such an arrangement proves feasible, it may be utilized in introducing the so-called moisture barriers into subgrades

and eliminate accumulation of water beneath pavements and frost damage.

The actual decrease in water content of a soil subjected to electro-osmosis is very small and thus, electro-osmosis alone is not economically feasible for large scale consolidation of soil masses. Casagrande (1949) has expressed the view that it may be possible to introduce a few electrodes at the bottom of the deposit and rely upon osmotic action to mobilize the full weight of the overlying soil and water for the purpose of consolidation. The possibilities of this technique have not apparently been explored as yet.

C H A P T E R I V

FLOW OF WATER UNDER THERMAL GRADIENTS

4.1. Introduction:

Ground temperature investigations show that the temperature in the region immediately below the surface varies with depth and with the daily and monthly fluctuations of the surface temperature, and that below a certain critical depth, the temperature is sensibly independent of the surface temperature fluctuations (Crawford and Legget, 1955). Above this critical depth, the daily fluctuations of the surface temperature give rise to a reversal of the direction of the temperature gradient every day; in countries where the monthly average temperature varies considerably from the annual mean temperature, the monthly fluctuations of the surface temperature also give rise to a reversal of the temperature gradient.

If substantial quantities of water may be shown to migrate in response to the prevalent temperature gradient, the implications of the above noted thermal gradients become

obvious: assuming, in accordance with the potential theory, that the quantity of water moved is proportional to the potential gradient causing flow and the duration of application of the gradient, a net accumulation of water on one side will be expected, if the durations of the two opposing temperature gradients are not equal. Field data reported by Benkelman and Williams^{*} reveal that the durations of the temperature gradients in a subgrade are indeed not equal. A typical set of data reported by these authors is reproduced here as an example; as will be seen from this plot, the temperature at the top of the base is higher than that at the bottom of the base, only for about 9 hours per day. Thus, there is ample justification for inquiring into the mechanism of moisture migration in soils due to thermal gradients and for ascertaining the extent to which such a migration may account for the observed accumulation of water beneath impervious pavements.

In the following sections, the available literature on this subject is reviewed in some considerable detail. The various quantitative methods of analyses are briefly described and evaluated. Finally, the present state of our knowledge of the mechanisms is summarized.

* "A co-operative study of Structural Design of Non-rigid Pavements", Highway Res. Board SR 46, 1959.

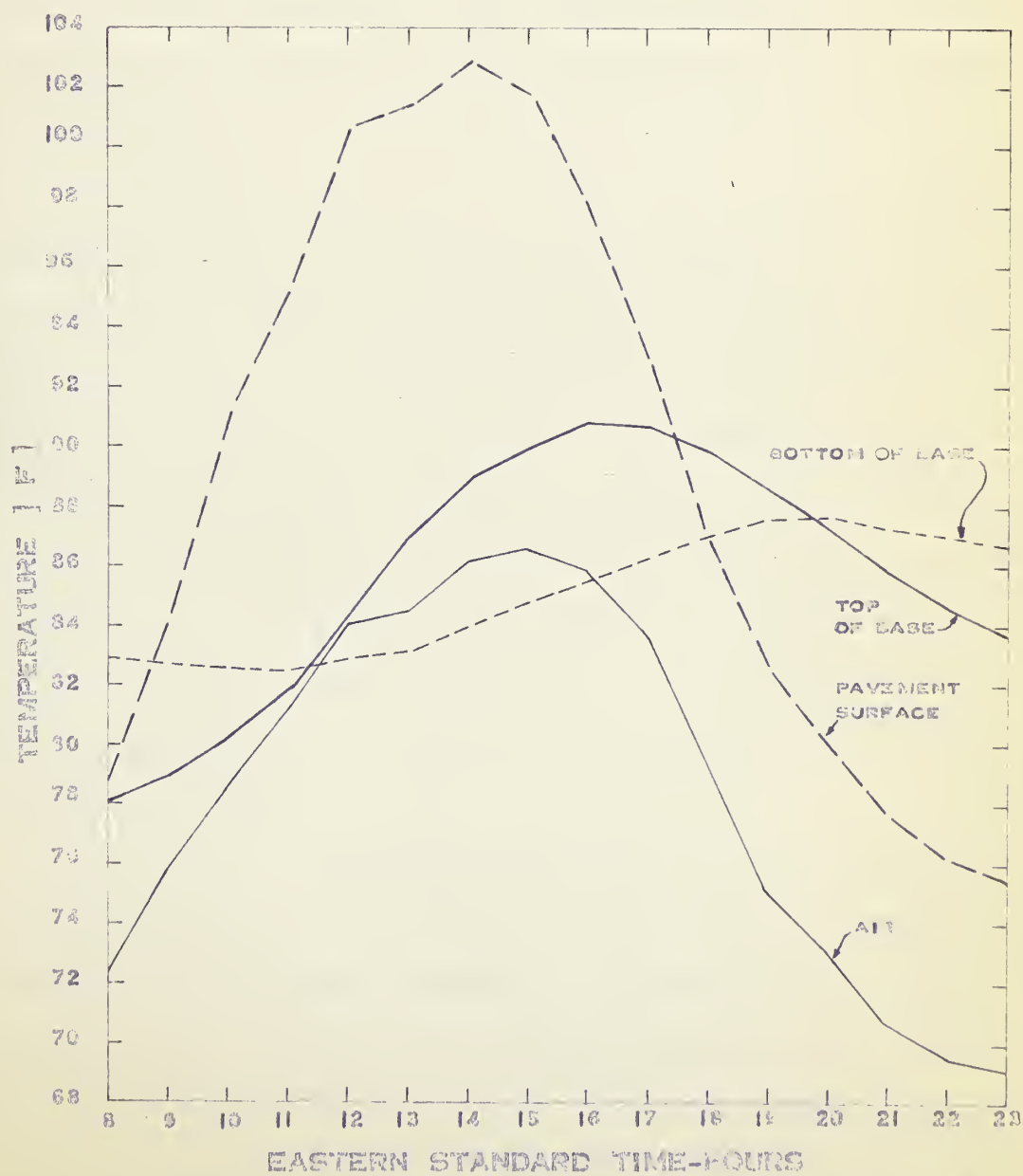
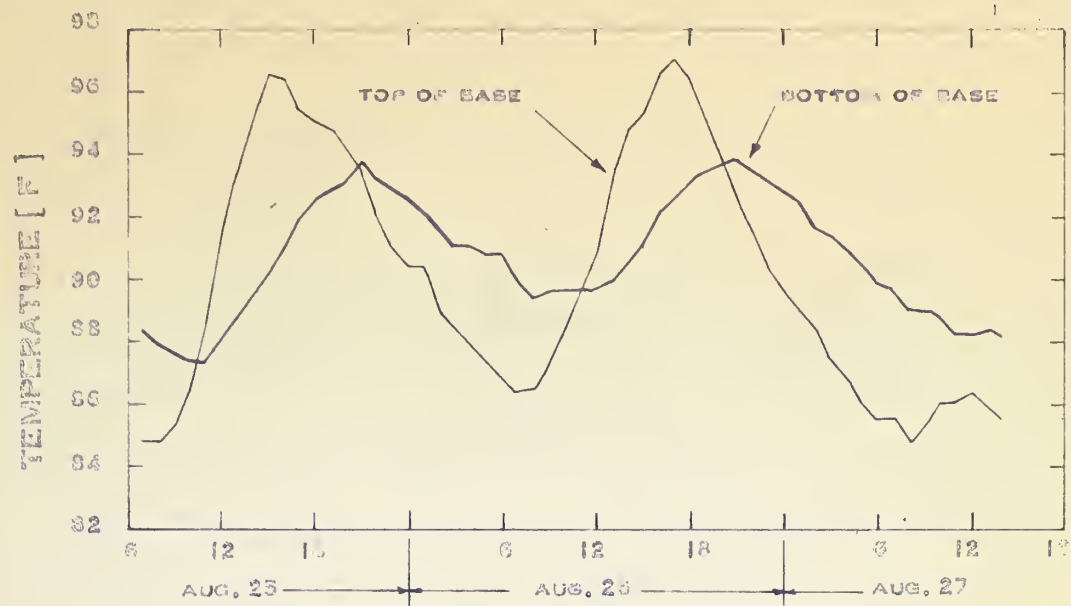


PLATE 1. TEMPERATURE AT 12 BASE COURSE SECTION [10 DAY AVERAGE]

4.2. Review of Literature

4.2.1. Historical^{*}

In 1907, Lippman observed that when a temperature gradient was applied across a membrane of gelatine separating two sections of a body of water, the water moved from the cold to the hot side, and that the observed rate of flow was about 50 mg. per minute for a temperature difference of 80°C; he called this the thermo-osmosis of liquids through porous membranes. Subsequently, Auburt, in 1912, found that the direction of flow depended upon the nature of the membrane: gelatine and pig's bladder gave rise to flow from the cold to the hot side, while parchment paper and viscose gave rise to flow in the reverse direction. He also found that porous plugs of inorganic materials gave rise to no flow; that the thermo-osmotic flow depended upon the presence of water-soluble substances in the membrane; and that the flow was not caused by the diffusion of these substances into the water on either side, but the flow took place only when these substance were still present in the membrane. These findings led Auburt to postulate that the phenomenon was electrical in origin and related to electro-osmosis.

In 1941, considerable quantities of flow of water, carbon tetrachloride and other liquids through sintered glass plates under a thermal gradient was reported by Derjaquin and Sidorenkov. This is at variance with Auburt's findings, who could detect no flow through inorganic materials.

^{*} The material for this section was largely drawn from the paper published by Hutchison et al. (1948).

Hutchison et al. (1948) conducted a series of experiments to check the data reported by Derjaquin and Sidorenkov and found that there was no flow through a sintered glass porous plate, if the plate and the liquid were chemically pure; flow occurred only when electrolytes were present in the porous plate. They suggested that the movement observed by the Russian investigators might be due to simple thermal expansion of the liquid or to the presence of impurities. These authors derived an equation for the thermo-osmotic pressure in terms of the heat of transfer, molar volume of the substance and the mean temperature; Winterkorn (1955) rederived this equation from a somewhat different line of reasoning, and utilized it in a Darcy type of equation for use in soil moisture studies.

No further work on pure thermo-osmosis has been reported. Thus, the available data reveal that thermo-osmosis does not occur through mineral (inorganic) porous substances, except under the presence of electrolytes in the membrane. As mentioned in an earlier chapter, soil grains usually have an ionosphere surrounding them, and so thermo-osmosis should be possible in soils, even though the solid constituents are purely inorganic in nature.

4.2.2. Thermo-osmosis in soils:

The effect of thermal gradients on soil moisture migration was reportedly studied by Bouyoucos as early as

1915^{*}. His extensive experiments revealed that there is a flow of moisture from the hot to the cold side in a closed soil column; that there is an optimum initial moisture content for maximum thermo-osmotic flow; and that this optimum moisture content falls in the vicinity of the plastic limit of the soil. Working with soil columns separated into sections by air gaps, Bouyoucos observed that the net transfer of moisture from the hot to the cold end was considerably reduced. From these findings, he concluded that the flow took place on account of the difference in capillary potential between the two ends, and that the flow was mainly in the liquid phase.

At about the same time, Lebedeff in U.S.S.R. observed substantial upward migration of water in field soils; after accounting for precipitation and evaporation, he computed that a volume of water equivalent to 66 mm. depth per unit area had migrated upwards from the ground water table during the winter 1914-15 (Lebedeff 1927). He ascribed this migration to the phenomenon of evaporation from the water table and condensation at the surface layers and implied that the flow was the result of diffusion of water vapour under a vapour pressure gradient.

* The original paper published by Bouyoucos was not available to the writer at the time of preparing this dissertation; Bouyoucos' work has been quoted by many of the subsequent investigators (see Winterkorn 1947, Rollins et al. 1954).

In 1939, Smith encountered the thermo-osmotic phenomenon when he was interested in the determination of thermal conductivities of moist soils. When the initial moisture content of the sample was such as to give 75% relative humidity for the soil atmosphere, the specimen appeared to be of uniform consistency after the application of a thermal gradient; when the relative humidity was raised to 99.3%, there appeared a distinct line of colour change, indicating a heterogeneous consistency in the final specimen. Smith investigated the matter further by determining the final moisture contents at three locations along the specimen for various soils and obtained results very similar to those obtained by Bouyoucos earlier. He reasoned that the changes in surface tension of the water induced by the applied thermal gradients were too small to account for the transfer, and that vapour diffusion under the imposed vapour pressure gradients was too slow a process to account for the rapidity with which equilibrium was obtained. He postulated that the phenomenon involved a mass convection of the water vapour across the pores (Smith 1939).

Subsequent investigations (Smith 1940) led Smith to alter his earlier ideas and postulate that the actual mechanism involved evaporation and recondensation of water in the soil pores nearest to the warm end, which caused a disturbance in the equilibrium of the capillary-held

water in the adjoining pores, thus causing migration to the cold side.

Winterkorn (1947) lists four main theoretical concepts that may be applied to thermo-osmotic phenomenon as follows:

1. diffusion of water vapour through the pore space under a concentration or partial pressure potential;
2. diffusion of water in solid solution, similar to the case observed for hydrophilic membranes separating chambers of different water vapour pressure;
3. movement as capillary water due to the difference in surface tension at different temperatures; and
4. flow in the film phase along the internal surface of the soil due to change in water affinity with change in temperature.

After discarding the first and the third of these concepts as of insignificant importance, he proposes that concept (4) should be applicable to soil-water systems and states" the exchangeable cations possess a greater activity (or are more dissociated) at the cold than at the warm side; they cannot move to the warm side because they are held by the negative charges of the mineral surfaces. The only way to decrease this concentration potential is by movement of water from the warm to the cold side". Based on this qualitative explanation, he developed a quantitative

method of computing the amount of thermo-osmotic movement in soils; the thermo-osmotic permeability computed in this manner had a value of 3.2×10^{-8} cm./sec. for a temperature gradient of 1°C per cm., while the experimental value was found to be 2.8×10^{-7} cm./sec. for the same conditions. Such a correspondence was considered to be close. In the same paper, Winterkorn made a brief mention about the detection of electric potentials between the hot and cold sides of the specimen and stated "the available evidence indicates that thermo-osmosis is a special form of electro-osmosis" (compare this with Auburt's findings mentioned earlier).

Jones and Kohnke (1952) studied the relation between water vapour diffusion and soil moisture tension (P_F) by applying a sharp temperature difference for 8 hours at the middle of soil columns contained in 6-inch test tubes and determining the final moisture profiles. From these experiments, these authors concluded that the movement of water vapour in the soils studied increased rapidly with the initial P_F and, after a certain level, the rate of movement dropped off sharply; and that the volume of unsaturated pores, and not their size, governed the transfer of water vapour. It is to be noted that these authors used a temperature gradient as a means of establishing a vapour pressure gradient in the specimens; as such, their investigations are not primarily concerned with thermo-osmosis;

nevertheless, their conclusion regarding the relative influence of the overall porosity and of the pore size is of much interest: if the porosity alone is the governing factor, soils having widely different pore sizes but the same porosity will be expected to have the same thermo-osmotic permeability (compare this with the similar condition obtaining in the case of electro-osmosis). Unfortunately, this factor has not been systematically investigated as yet.

Gurr, Marshall and Hutton (1952) reported a series of experiments on thermo-osmosis of water through soils with widely different initial moisture contents. These authors attempted to assess the extent of movement in the liquid and vapour phases by tracing the movement of small quantities of salt added to the soil prior to the application of the thermal gradient. All samples were subjected to a temperature gradient for a constant period of 5 days, and then the salt content and water content of the various sections were determined. In all except the "driest and wettest" columns, there was a net transfer of water to the cold side and of salt to the hot side. The authors concluded that the thermo-osmotic transport of water occurred in the vapour phase from the hot to the cold side of the specimen and that the resulting accumulation of water at the cold end caused a pressure gradient in the liquid phase in the reverse direction, giving rise to a return flow of water in the liquid phase to the hot end.

Taylor and Cavazza (1954) reported results of a series of tests on thermo-osmosis of water through a silty loam soil: they introduced air gaps varying from 1 mm. to 10 mm. widths between adjacent sections of the specimen and applied temperature gradients varying between 1.5 to 1.7°C/cm. They found that a stable (equilibrium) distribution of moisture could be produced in a continuous specimen after a sufficient period of application of the temperature gradient while such a dynamic equilibrium state was not obtained during the same period in the segmented specimen. They reasoned that the return flow in the liquid phase was responsible for the establishment of steady conditions in the former case, and the absence of it caused a continuous movement of vapour towards the cold side in the latter case. The writer feels that had the experiments been continued for longer periods, a steady equilibrium condition would have been produced in the segmented column also, since the moisture contents of the various segments would tend to attain equilibrium with the prevalent vapour pressure and temperature conditions.

Rollins, Spangler and Kirkham (1954) constructed an apparatus in such a way that the water accumulating at the cold end could be transferred to the hot end of the specimen through an external capillary tube; a drop of mercury placed in the external capillary tube served as a means of indicating the direction of flow; the rate of

movement of this drop could also be taken as an indication of the rate of thermo-osmotic flow. They observed that there was no tendency for the drop to move from the warm side to the cold side of the specimen and concluded that liquid flow from the hot end to the cold end could not be of significance. With open systems (water movement through the external capillary tube permitted) they observed that the accumulation of water at the cold end was not appreciable, while there was considerable accumulation in a closed, non-circulating system. Thus, their experiments confirmed the earlier hypotheses of Gurr et al. (1952) and of Taylor and Cavazza (1954).

The effect of density and air content of the samples on the thermo-osmotic flow rates were also investigated. As would be expected from a qualitative analysis, the flow rate was found to increase with increasing air contents and decreasing densities.

The influence of boundary temperatures and of the temperature gradient on the flow rate was studied. While the flow rate increased with increasing temperature gradients, the variation was not linear. Such a trend needs further verification and corroboration; if the non-linear relationship between the flow rate and the temperature gradient is definitely established, use of potential theory to describe the thermo-osmotic phenomena may be open to

question, since the theory assumes a constant proportionality factor between the flux and the potential gradient.

Hutcheon (1955) postulated a qualitative line of reasoning to account for the existence of an optimum moisture content for maximum thermo-osmotic flow. This line of argument is based on the hypotheses of Gurr et al., and of Taylor and Cavazza, that a return flow of liquid may take place from the cold to hot end under certain conditions of moisture content.

Swenson and Sereda (1955) described the construction of an apparatus in which movement of water through soils, concrete and similar porous materials could be studied keeping the moisture content of the specimen constant; this was achieved by maintaining suitable suction pressures across porous plates in contact with the ends of the sample. Results of preliminary experiments on Ottawa clay confirmed the findings of earlier workers. The authors state " ... at this stage it is not at all evident what contribution is made by the existence of the interface between the sample and the porous plate. This interface might even make the main contribution to the observed flow. Until this is assessed by further experimental work, this investigation must remain as a proposed method to study the phenomenon of moisture movement due to a temperature gradient".

Winterkorn (1955a) presents experimental and theoretical evidence to show the strongly electric character of the moist soils and argues that if electrical forces could produce mechanical effects in soil-water systems (as evidenced, for example, by electro-osmotic drainage of soils), thermal gradients should be logically expected to have electrical consequences. In support of this deductive reasoning, he cites the data obtained by Gowda and later by Williams, both of whom studied the generation of electric potentials during thermo-osmotic flow through natural and several homoionic modifications of "New Jersey Hagerstown soils", H-bentonite, H-kaolinite and H-grundite. Fairly high temperature gradients (2.84 to $4.03^{\circ}\text{C}/\text{cm.}$) were used and the tests were run for periods not exceeding 100 minutes. According to the author, "since the rate at which equilibrium is approached varies as a function of the distance from the equilibrium, such data (obtained with longer durations of testing) cannot serve as a good basis for the calculation of thermo-osmotic transmission coefficients ...".

In all experiments, electric potentials were measured and it was observed that the establishment of an electric potential was "almost instantaneous with application of hot and cold plates", and did not await the establishment of a uniform temperature gradient. The observed electric potentials varied from 300 to 700 millivolts; the polarity

of the thermo-electric potential for 'normal' soils was the same as that of applied potential to cause migration of water in the same direction; the polarity was reversed in the case of 'refractory' Al- and Mg- soils.

As for the mechanism of thermo-osmotic transfer, the movement is ascribed to vapour flow in the case of dry soils (appreciably drier than the plastic limit) and to liquid flow in the film phase in the presence of continuous adsorbed films..

In a subsequent paper (1955b) Winterkorn derives an equation for the pressure or suction associated with the transfer of one gram of water from a region of temperature, T , to another at $T + dT$; the final equation, based on the second law of thermodynamics, is of the form:

$$dP = \frac{Q}{l} \times \frac{dT}{T}, \quad (1)$$

where P is the maximum pressure difference associated with a temperature gradient dT/dl ,

Q is the latent heat involved in the change of restraint of one gram of water incident to the same temperature gradient, and the numeral 'unity' represents the specific volume of water. Substituting this value in a Darcy type equation, Winterkorn obtains a thermo-osmotic flow equation.

Eq. (1) has been derived from a different view point earlier by Hutchison et al. (1948), although these authors were concerned with the thermo-osmosis through membranes and so the 'latent heat' becomes the 'heat of solution' of the

the first of these is the fact that the system is not self-
sufficient. It is dependent on the external world for its
operation. This is a serious defect, and it is one which
must be remedied.

The second defect is that the system is not self-
correcting. It is dependent on the external world for its
operation. This is a serious defect, and it is one which
must be remedied. The third defect is that the system is
not self-sufficient. It is dependent on the external world for
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operation. This is a serious defect, and it is one which
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not self-sufficient. It is dependent on the external world for
its operation. This is a serious defect, and it is one which
must be remedied.

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correcting. It is dependent on the external world for its
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must be remedied. The ninth defect is that the system is
not self-sufficient. It is dependent on the external world for
its operation. This is a serious defect, and it is one which
must be remedied.

The tenth defect is that the system is not self-
correcting. It is dependent on the external world for its
operation. This is a serious defect, and it is one which
must be remedied. The eleventh defect is that the system is
not self-sufficient. It is dependent on the external world for
its operation. This is a serious defect, and it is one which
must be remedied.

liquid on the membrane. But the use of this equation in a Darcy type law, as is done by Winterkorn, and the eventual formulation of a mathematical relationship between the thermo-osmotic permeability and the hydraulic (Darcy's coefficient of) permeability appear to be gross oversimplifications of the entire phenomenon. No experimental support of this relationship has been adduced, either by Winterkorn himself, or by other investigators. On the contrary, data furnished by Rollins et al (1954) reveal that the volume rate of flow under thermal gradients depends upon the temperature gradient, as well as upon the boundary temperatures; in such case, the simple potential theory - as embodied in Darcy's law - cannot be assumed to hold.

Habib (1957) reported some experiments on thermal transfer of moisture in saturated and unsaturated soils; fully saturated specimens were tested both as open and closed systems separately. A continuous flow of water in the former case and the establishment of positive pore pressures in the latter case were observed. The magnitudes of the reported pore pressures are very small, not exceeding 50 gm./sq. cm., but according to the author, "the pressure differences (closed system) and the flows (open system) are multiplied by three or four when the interstitial water contains an ionizable salt". No reasons for this rather

significant effect of salts are furnished by the author; data on the observed flow rates have also not been reported. The writer is inclined to believe that the rates could not have been significant and might be attributable to flow under a surface tension gradient.

Kuzmak and Sereda (1958) made use of the special apparatus developed earlier by Swenson and Sereda (1955) (wherein flow of water through porous materials could be maintained without altering the moisture content of the specimen) and studied the movement of water under temperature gradients through porous plates of different textures separated by gaps of different widths. They found that a flow occurred only when the suction pressure in the gap was increased to such a value as to desaturate the same, thus confirming the concept of vapour flow under a thermal gradient. Salt tracer experiments with a sample of fine sand in the apparatus revealed that the observed transfer of salt from the warm to the cold side could be completely accounted for by the isothermal diffusion of salt. Measured streaming potentials during thermo-osmotic flow were in general found to be the same as the values observed under conditions of no flow. These two observations lent support to the earlier conclusion and thus the authors concluded that vapour flow was the chief mechanism involved.

The writer feels that a more sensitive test of the concept could have been obtained by using salt solutions of equal solute concentrations on both sides, thus eliminating isothermal diffusion.

Hutcheon (1958) reports very extensive experimental data on thermo-osmotic flow of water under small temperature gradients (0.5 to 0.8°C/cm.); in some of his experiments, the cold end was maintained at -4°C, thus freezing the accumulated water and preventing a return flow of liquid from the cold to the hot side. To account for the moisture profiles obtained, he postulates that a return liquid flow must exist even in the warm section of the soil column, transferring water from the interior sections to the warmest place, that is, the 'evaporation sink'.

4.2.3. Quantitative methods of analysis:

DeVries (1950) assumed the validity of an equation of the form:

$$q = -\alpha \cdot a \cdot D \cdot \gamma_w \cdot \frac{P}{P-\eta} \cdot \frac{\partial \eta}{\partial x} \quad (2)$$

for unidimensional molecular diffusion of water vapour through soils. The various symbols in the above equation were defined as follows:

q = vapour flux in g/cm² sec.,

α = a function of the soil structure, dimensionless,

a = volume fraction of gas filled pores,

D = diffusion coefficient of water vapour in air, cm²/sec.,

P = total pressure of moist air 760 mm. Hg.,

p = partial pressure of water vapour in mm. Hg.,

x = coordinate distance, in cm.

Penman (1940) gave the empirical value $\alpha = 0.66$. The value of the diffusion coefficient was assumed to be given by

$$D = 4.42 \times 10^{-4} \frac{T^{2.3}}{P} \quad (3)$$

where T = the absolute temperature in $^{\circ}\text{K}$.

When this equation was applied to predict the movement of water under temperature gradient, it was found that the observed quantity of flow (or the rate of diffusion) was considerably greater than the calculated amount, the ratio of the two rates varying from three to eighteen (Hutcheon 1958, Philip and deVries, 1957). This discrepancy between the diffusion theory and the experimental data was ascribed to the possible presence of molecular slip flow, or to the interaction between the water vapour and the soil (Penman's coefficient of 0.66 was based on experiments with carbon dioxide and acetone, neither of which presumably reacted with the solid or liquid phase of the medium; see Philip and deVries, 1957).

A very plausible explanation of the observed discrepancy is furnished by the interesting experiments of Woodside and Kuzmak (1958). These authors constructed an enlarged 'model' of the solid grain-pore system: 6 inch diameter marble spheres were used to represent the soil

grains, bitumen to represent the interstitial water and silica aerogel to represent the soil air; the materials were so chosen as to give thermal conductivity ratios comparable to those of the soil-water-air system. They found that the temperature gradient across an air filled pore space in a solid-air system was about six times the average overall temperature gradient; though no actual measurements were carried out, they express the opinion that the ratio will be less than six in a soil-water-air system and will increase with decreasing water contents. In the light of these findings, these authors propose that movement of water under a temperature gradient "involves multiple vapour diffusion and liquid flow steps in series" and that, since the resistance to liquid flow through the contact wedges is negligible, the rate of transfer is governed by the rate of diffusion across the pore space. The validity of these experiments and of the conclusions derived therefrom have been seriously questioned later by deVries and Philip (1959).

Philip and deVries (1957) have developed a mathematical theory to describe thermo-osmosis, recognizing that the transfer involves a "series-parallel flow" through air-filled pores and contact wedges; the vapour flux is assumed to be governed by the vapour pressure gradient across the air-filled pores, and the liquid flux, on account

of the negligibly small resistance, is assumed to adjust itself to the vapour flux. The final equation is of the form:

$$q = (a + \theta) D \cdot \frac{P}{P - P_s} \cdot \beta (\nabla T)_a \quad (4)$$

where, in addition to the symbols already defined,

θ = volumetric moisture content of the soil,

$\beta = \frac{dP}{dT}_0$ = the rate of change of density of saturated water vapour, due to change in absolute temperature, and

$(\nabla T)_a$ = the temperature gradient across the air-filled pores.

Applying this equation to the experimental data reported by Rollins et al. (1954) and Gurr et al. (1952), the authors find that the agreement is "quite satisfactory in view of the approximate character of the analysis".

4.3. Present Status of Our Knowledge

4.3.1. Concerning the phenomenon:

A survey of the available literature given above, reveals that the existence of a thermo-osmotic flow through soils has been definitely established. All previous investigators have, in general observed

- (1) that there is a transfer of water from the hot to the cold side of a specimen;
- (2) that the quantity of water transported is a minimum in very dry and very wet soils;
- (3) that there exists an optimum initial moisture content for maximum thermo-osmotic flow; and
- (4) that, in fairly moist soils, there may be a return flow

of water from the cold to the hot side of the specimen. These observations are for partially saturated soils only. The only set of experimental data on saturated soils is that reported by Habib; the values of pore pressures in a closed system observed by him are very small for most practical purposes; flow rates in open systems have not been reported.

4.3.2. Concerning the mechanism of thermo-osmosis:

There appears to be a considerable amount of controversy as to the nature of the actual mechanism involved in the phenomenon. In general, five distinctly different mechanisms have been proposed by various workers:

- (1) liquid flow under a thermally induced capillary potential (suction) gradient (Bouyoucos);
- (2) evaporation and recondensation of water vapour in the hot end which gives rise to a liquid flow in the colder segments due to capillary potential gradient (Smith);
- (3) osmotic flow in the film phase due to thermally induced differential water affinity of the soil (Winterkorn);
- (4) vapour diffusion to the cold side followed by a return circulation in the liquid form under favourable conditions (Gurr et al., Taylor and Cavazza, Hutcheon and others); and
- (5) multiple vapour diffusion and liquid flow steps in series (Woodside and Kuzmak), and a series-parallel flow of vapour and liquid (Philip and deVries).

It is very difficult to state definitely which of these mechanisms is the most probable one; obviously it is quite possible for all of them to be involved at the same time, each one making a more or less contribution to the total flow. The critical question to be resolved is: what are the relative contributions of each of these mechanisms under different conditions of soil moisture content? It is not possible to give a quantitative answer to this question at the present stage of the development; numerous experiments, conducted under widely different conditions, will be required before a tentative answer could be proposed.

4.3.3. Concerning the quantitative methods of analysis:

The inadequacy of the simple molecular diffusion theory has already been mentioned. The extended analysis proposed by Philip and deVries (1957) has not been systematically verified yet. Even so, the analysis is for the case of dry media only, where there is no continuity in the liquid phase; for the moist media with liquid continuity, they propose empirical coefficients to account for the decreased volume of the pore space available for vapour flow. Finally, none of these methods of analysis recognize the existence of a return flow of liquid from the cold to the hot side; as such, they can be used only for open systems where recirculation is prevented.

4.3.4. Concerning the physical explanation of the phenomenon:

A valid physical explanation of the phenomenon will, of course, depend upon the nature of the mechanism assumed to be active. As already seen, Winterkorn (1947) believes that the flow is in the adsorbed films of water along the internal capillaries of the soil; he ascribes the cause of this movement to the greater water affinity of soils at lower temperatures than at higher temperatures. In support of this hypothesis, he cites thermo-osmotic experiments on an extracted clay where consistent electric potentials were observed between the ends of the specimen.

As already mentioned, when a liquid is forced through a fine capillary passage, a potential gradient (streaming potential) is set up between the ends of the capillary, tending to oppose the motion. Thus, the existence of an electric potential during thermo-osmosis may be taken as a strong indication of the presence of a flow in the liquid phase. However, Kuzmak and Sereda (1958), working with sandy soils, report that no streaming potentials were observed.

The other physical explanation is that advanced by Hutcheon (1958), based on the 'vapour diffusion-recirculation in the liquid phase' concept. Briefly, his arguments proceed as follows: In very dry soils, the rate of change of relative humidity (and hence the vapour pressure) of the soil air due to changes in moisture content is very high;

consequently, small changes in moisture contents are sufficient to equalize the thermally-induced vapour pressure gradients. The total transfer is hence small, and a recirculation in the liquid phase is impeded by the very low unsaturated permeability of the soil. As the initial moisture content increases, the rate of change of relative humidity falls off, thus necessitating a larger change in moisture content to equalize the same vapour pressure gradient; at the same time, the air-filled porosity is decreasing, thus decreasing the rate of dissipation of the vapour pressure gradient. At the cold end, the rate of condensation and consequently the rate of building up of a suction gradient are increasing, due to the increased relative humidity of the soil atmosphere.

Thus the system now involves two effects: with increasing moisture contents, the rate of dissipation of the vapour pressure gradient decreases, while the rate of building up of a suction gradient from the cold to the hot side, and the unsaturated permeability of the soil are increasing. It is apparent that there must exist a particular moisture content at which these two opposing tendencies will be so balanced, as to give a maximum net transfer of water from the hot to the cold side. At very high moisture contents, the air filled pores become discontinuous and so vapour diffusion ceases. The only movement, under these

circumstances, will be in the liquid phase under the thermally induced suction gradients, and the magnitudes involved are very small.

The above arguments (presented in a slightly modified and abbreviated form) appear to be sound and plausible; above all, they clearly demonstrate the existence of an optimum moisture content for maximum thermo-osmotic flow. But, if the primary flow is in the vapour phase, it will be very difficult to explain how an electric potential is created across the specimen during thermo-osmosis: it is not established - to the writer's knowledge - whether liquid-gas interfaces may also give rise to electrokinetic phenomena, similar to solid-liquid interfaces; if they do not, then, a vapour flow relative to a liquid boundary cannot cause a streaming potential to be created. This argument presupposes that a streaming potential is always observed during thermo-osmosis; so far, the data reported are contradictory in this respect. Winterkorn and his co-workers observed and measured consistent values of electric potentials with clayey soils, while Habib, working with "mud samples", and Kuzmak and Sereda, working with "fine sand", could not find any consistent voltage or polarity across the samples. Thus, the question is still unsettled.

4.3.5. Concerning similarities between electro-osmosis and thermo-osmosis:

Auburt's findings (see Hutchison et al. 1948) led him to postulate that the phenomenon of thermo-osmosis is electrical in origin and related to electro-osmosis. Subsequently, Winterkorn (1947, 1955a) independently arrived at the same conclusion, from a line of reasoning, entirely different from the one employed by Auburt earlier. He also found that an electric potential was created across the ends of a specimen of clayey soil subjected to thermal gradients, and that the establishment of this electric potential was almost instantaneous. It is true that subsequent investigators (Habib 1957, Kuzmak and Sereda 1958) have been unable to detect any consistent electric potentials during thermo-osmosis, but, this need not necessarily exclude the possibility of the presence of electric potentials; in other words, owing to the type of soil used by the later investigators, the induced electric potentials may have been too small to be measurable by ordinary methods.

Jones and Kohnke (1952) have stated that the volume of air-filled pores and not the size of the pore governs the vapour transfer in soils. As mentioned in an earlier section, this condition is very similar to the one obtaining in electro-osmosis, where the discharge is governed by the porosity of the soil and not by the individual pore size.

Thus, the available evidence indicates that it is distinctly possible that electro-osmosis and thermo-osmosis are related in some as yet unknown manner.

C H A P T E R V

EXPERIMENTS ON THERMO-OSMOTIC FLOW IN SOIL

5.1. Scope of the Investigations:

Previous investigators have assumed that in relatively moist soils, thermo-osmotic flow gives rise to a return flow of liquid water from the cold to the hot side (Gurr et al. 1952, Taylor and Cavazza 1954), and Hutcheon (1955, 1958) has proposed a physical explanation of the overall phenomenon accordingly. The present investigations are aimed at determining whether such a flow takes place or not, thus verifying the hypothesis advanced by Hutcheon.

5.2. Experimental Arrangement

5.2.1. Preliminary considerations:

If the soil moisture, transferred to the cold side by thermo-osmosis, tends to redistribute itself under a suction gradient, it is obvious that the moisture profile of the specimen should be time-dependent during the early stages of the flow. Thus, the experimental set-up should

be so devised as to subject several specimens of identical initial conditions to varying durations of application of a constant temperature gradient. A somewhat similar testing program was employed by Taylor and Cavazza when they studied the moisture profiles of continuous and segmented columns of soil, with a view to differentiate between liquid flow and vapour flow; however, these authors used only one initial moisture content that gave an equilibrium profile in the continuous columns, at the end of seven days. In the present series of investigations, six different initial moisture contents were employed. The apparatus used in this investigation is broadly similar to the one employed by Hutcheon (1958) in his investigations.

5.2.2. Soil sample:

About 85 lbs. of the sample were obtained by grinding pure silica sand so that the final material will completely pass a 60-mesh sieve. Attempts to obtain a finer grinding proved to be uneconomical and time-consuming on account of the large quantities of sample involved and of the extremely high resistance of the soil.

Subsequent to grinding, the sample was digested with excess dilute hydrochloric acid for a period of 72 hours so as to remove the iron as ferric chloride; this was followed by repeated washing with distilled water to remove the salt. A vacuum technique was used to filter the distilled water out of the sample. After washing, the sample was over-dried at $105 \pm 5^{\circ}\text{C}$.

UNIVERSITY of ALBERTA
DEPT. of CIVIL ENGINEERING
SOIL MECHANICS LABORATORY
GRAIN SIZE CURVE

PROJECT

SITE

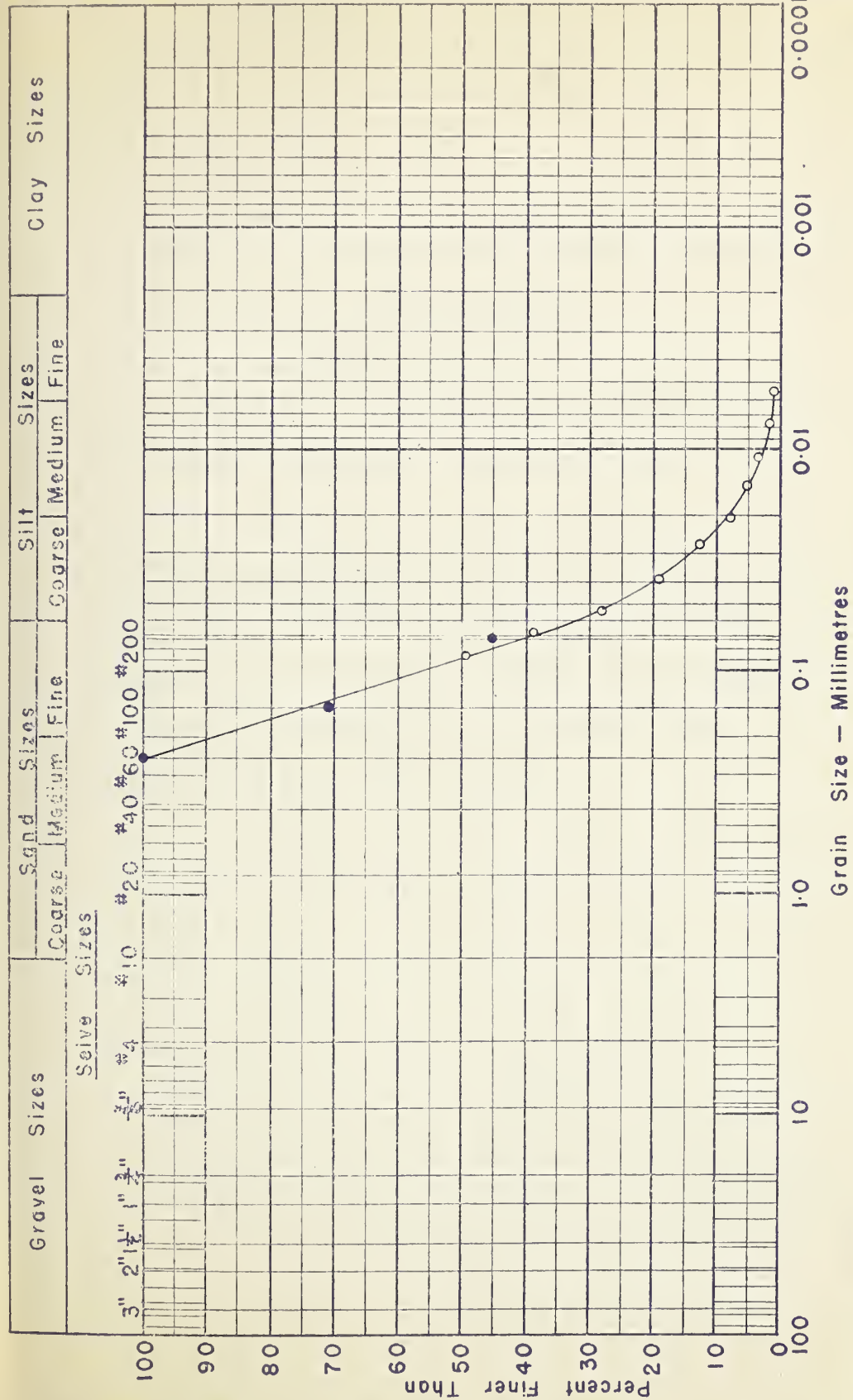
SAMPLE

LOCATION

HOLE

TECHNICIAN

DATE 25.8.60


$$D_{10} = \frac{0.022}{5} \text{ mm.}$$

Remarks: WET SIEVE ANALYSIS

0 HYDROMETER ANALYSIS

PLATE 2.

PLATE 2

5.2.3. Soil container:

Commercially made 44 mm. "Lucite" tubes were employed as specimen containers; the tubes were cut into rings 2 cm. long, and eight such rings were taped together by adhesive tape to provide a container for each specimen, thus yielding specimens 16 cm. long x 4.4 cm. diameter. Necessary holes were drilled on the sides of these rings for introducing thermo-couples.

5.2.4. End plugs:

The end plugs were made of open-ended brass rings that would fit snugly into the container ring. The end closest to the specimen was covered by an $\frac{1}{16}$ " brass disc soldered in position; the other end was closed by a similar plate with two tubes projecting out. Thus, the end plugs could be used as thermal contacts, by circulating water at the desired temperature through them.

5.2.5. Hot and cold sources:

Two $\frac{1}{16}$ " G.I. tanks, 1'9" x 1'9" x 1'9", with individual capacities of approximately 34 gallons, served to hold the hot and cold water to be circulated through the end plugs of the specimens. An ordinary laboratory freezing unit, operating in conjunction with a sensitive bi-metallic thermo-regulator was employed for maintaining constant cold-end temperature. A thermostatically controlled (as claimed by the manufacturer) domestic heating coil was

used in the hot side; soon it was found necessary to have an auxiliary thermo-regulator on this side as well, and so another bi-metallic thermo-regulator was connected in series with the heating coil.

5.2.6. Circulation system:

Water from the hot and cold tanks was circulated through the respective end plugs of the specimens by means of two water-throw pumps; all specimens were arranged in parallel (see schematic diagram of the lay-out). Initially, the pumps were so arranged as to suck water from the tank, force it through the battery of specimens and then discharge the same back into the tank. Preliminary tests revealed that the temperature of the water was raised by 2 to 3°F during the passage through the pump, presumably due to frictional effects. In order to minimize the temperature difference between the tank and the ends of the specimen, the pumps were rearranged so as to suck water from the battery of specimens and deliver it into the tank. In this manner, water could flow through the end plugs of the specimens under the combined influence of the hydrostatic pressure of the body of water in the tank and the suction pressure exerted by the pump; this, together with the smaller length of discharge pipe, facilitated an increased discharge while eliminating the difference in temperature between the tank and the ends of the specimens (Plate 1(a)).

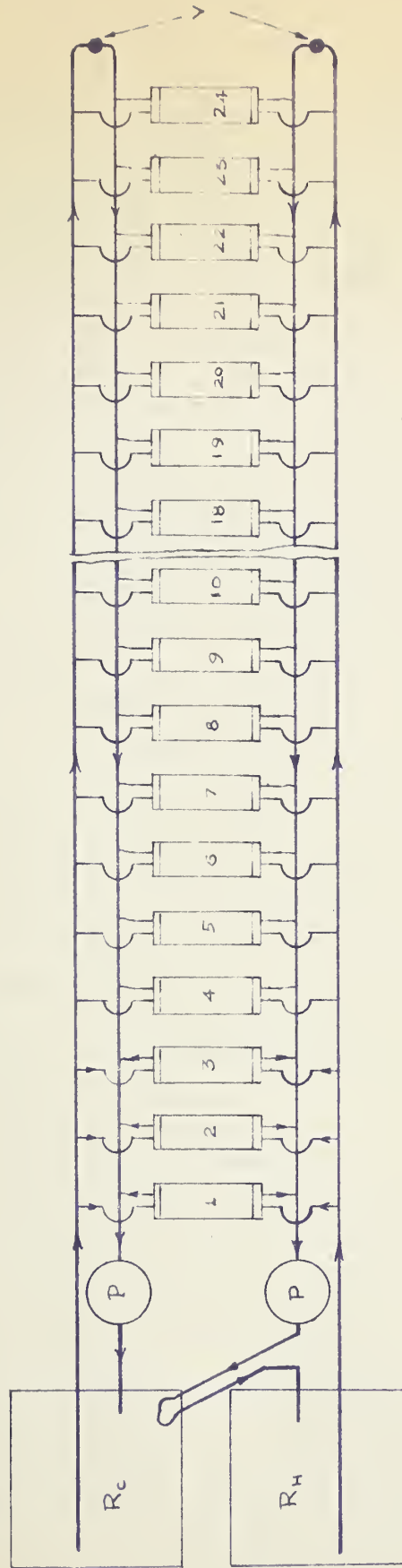


Fig 1(a)

SCHEMATIC ARRANGEMENT OF THE APPARATUS

R_C - RESERVOIR FOR COLD WATER.

R_H - RESERVOIR FOR HOT WATER.

P - PUMPS.

V - REGULATOR VALVES.

1, 2, ... SPECIMENS.

ARROWS INDICATES DIRECTION OF FLOW OF WATER
ELECTRICAL CONNECTIONS AND THERMO-COUPLES NOT SHOWN.

At the cold end, the heat generated by the passage of water through the system was taken away by the refrigerator unit automatically. Since there was no such heat-absorbing unit at the hot end, a small length of thin copper ($\frac{1}{2}$ " dia.) tube was interposed between the discharge end of the pump and the outlet in the hot tank; this piece of copper tube was kept immersed in the cold tank so that the water flowing through would be cooled down; the temperature was again raised to the desired value by the heating element in the hot tank. The very large volume of water in both tanks (about 30 gallons each, compared to the rate of pumping which did not exceed 3 gallons per minute) acted as a buffer in minimizing the effect of the quantity of heat on the temperature of the body of water. In the final arrangement, the individual temperatures of water in both tanks did not vary by more than $\pm 1^{\circ}\text{F.}$, with the cold end temperature varying within even narrower limits.

5.2.7. Thermal insulation:

The two tanks were completely insulated by commercially-made, 3" thick cotton wool pads. The battery of specimens, together with the pipe lines were placed inside a specially constructed plywood box (approximately 12'0" x 2'0" x 0'8") and the box was filled with "zonolite" completely, thus providing adequate thermal insulation for the system.

5.3. Experimental Procedures

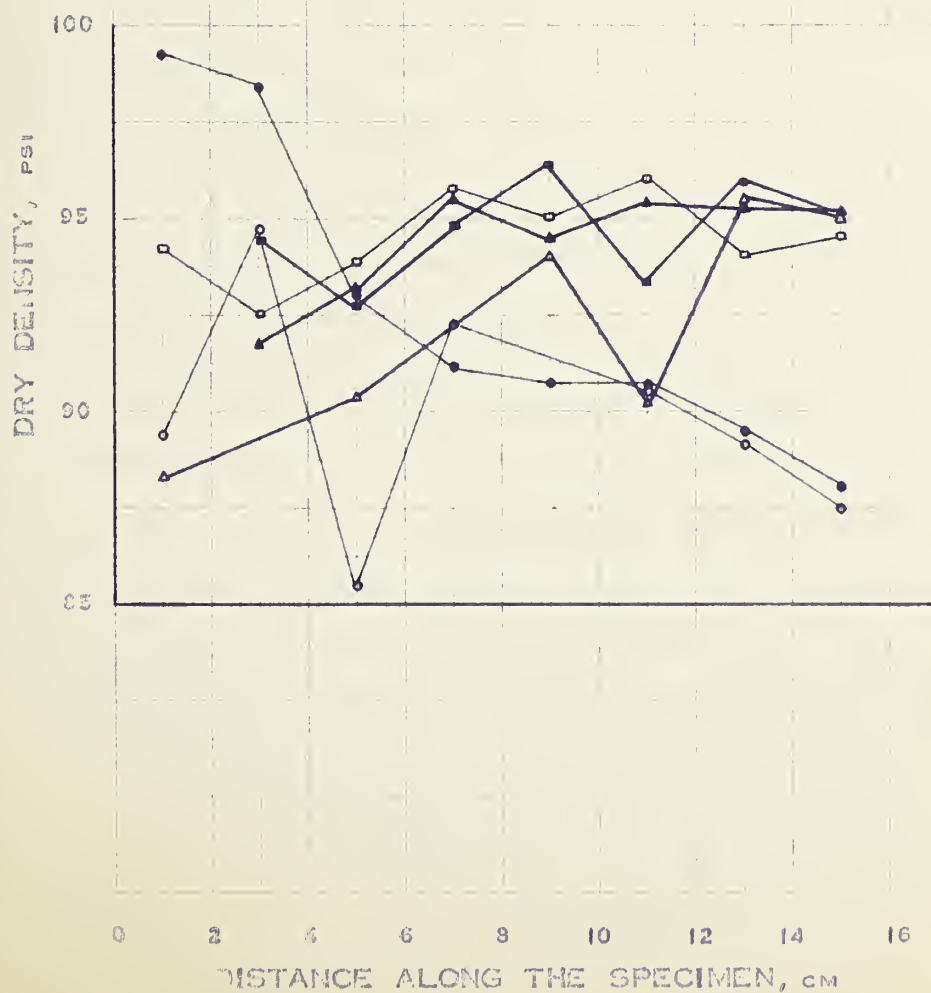
5.3.1. Compaction of specimens:

With the desire to achieve a highly uniform and reproducible degree of compaction, various conventional and novel methods were tested: these methods included (1) dynamic compaction (similar to conventional Proctor's method), (2) static compression from one end, (similar to Porter's method), (3) static compression from both ends, and (4) static axial compression aided by lateral vibratory blows delivered manually and with a high frequency electric hammer. Finally, static compression from both ends, aided by lateral vibratory blows with an electric hammer was chosen as the technique to be used; even in this case, the dry densities of the various sections of the specimen were found to vary erratically over a range of ± 3 pcf about an average mean density of 95 pcf. The random nature of the variation of densities, together with the fact that the experimental determinations involved a probable error of ± 2.5 pcf in the dry density values, indicated that a better degree of uniformity could not be obtained.

Before the start of the experiments, about 500 gm. of the dry soil was mixed thoroughly with enough quantity of distilled water to give the desired moisture content and was placed in a sealed water-proofed cardboard cylinder. All these cylinders were left in the humid room for a period

PLATE 3
COMPARISON BETWEEN VARIOUS COMPACTION TECHNIQUES

- — DYNAMIC COMPACTION WITH A FALLING RAMMER
- — STATIC COMPRESSION
- △ — STATIC COMPRESSION AIDED BY VIBRATORY LATERAL
BLOWS, MANUALLY DELIVERED, AXIAL PRESSURE
30 KG/CM².
- ▲ — SAME AS ABOVE, AXIAL PRESSURE 5 KG/CM².
- — VIBRATION WITH AN ELECTRIC HAMMER WITH AN
AXIAL PRESSURE OF 5 KG/CM².
- — SAME AS ABOVE, AXIAL PRESSURE 30 KG/CM².



of at least 72 hours, at the end of which period, the cylinders were re-opened, the soil was mixed again and the necessary quantity of the wet soil to give the desired dry density was weighed into the assembled mold. The remaining wet soil was used for determining the initial moisture content by oven-drying.

After compaction, the specimen (inside the "Lucite" container) was removed from the mold, covered tightly with aluminum foil and was left in the humid room till it was ready to be mounted in the apparatus. In all, six different initial moisture contents were used with five specimens each for the first run (temperature gradient 38 to 110°F.) and four specimens each for the second run (42 to 78°F.). (The apparatus was designed to accomodate twenty four specimens at any one time).

5.3.2. Assembling the specimens in position:

A 3" layer of "Zonolite" was first laid inside the plywood box and the brass end plugs were assembled in their respective positions. (The inlet and outlet tubes of the brass plugs were connected to the main pipes by means of short lengths of "Tygon" tubing). An 8" wide plank was laid on top of the insulation and was adjusted to be truly horizontal. The specimens, with their ends trimmed plane, were then mounted in position between the end plugs; the plugs and the specimens were bound together with adhesive

tape. Necessary thermo-couple leads were then inserted into the specimen at predetermined locations and then the box was completely filled with "Zonolite".

5.3.3. Actual experiment:

After assembling the apparatus, circulation of water through the end plugs was started and periodic temperature readings were taken, both of the water in the tanks and of the different locations along the various specimens.

Due to experimental difficulties, the first specimen of the first run of tests could not be taken out for moisture profile determination before an elapsed time of 77 hours. In view of the fact that the tests were concerned with changes in the moisture profiles at different times, it was decided to retain this period as the same for all the runs. Subsequent specimens of each run were taken out at 24 hour intervals.

As soon as a specimen was taken out, it was clamped between two flat plates held together in an universal soil extruder. (A bench vise could have been used in the place of the soil extruder, had the specimen been much shorter). The individual rings were cut out by means of a razor blade and a wire saw and the weights were recorded. The moisture content of the soil in each ring was obtained by oven-drying the wet soil for a period of about 15 hours; the weight and diameter of the

rings were measured and recorded. All weight measurements were carried out in a semi-automatic balance correct to 0.0001 gm., and the diameter was measured with a specially made tapered micrometer, correct to ± 0.001 in.

During the course of the experiments, the temperatures at various sections of individual specimens were measured at periodic intervals using iron-constantan thermo-couples. With a view to minimize the errors in the computed values of density and void ratio of the various sections of the specimens, the number of thermo-couples was kept as small as possible. In general, the temperatures at homologous points of the various specimens showed no tendency to vary regularly with the distance along the apparatus; this will be seen from the typical data sheet shown on page 98.

5.3.3. Computations:

The individual dry densities of the various segments of the specimen were used to compute the corresponding void ratios, and then, in conjunction with the computed average moisture content of the soil within the ring, the degrees of saturation were computed. The degree of saturation, rather than the moisture content, was chosen as the means of presentation of the results, on account of the inevitable variations in dry density (and hence void ratio) across the specimen.

In some cases, especially in the drier specimens, dry densities could not be determined for all of the individual segments, since the dry soil tended to slough off; in all these cases, the moisture content of the soil in the warmer sections of the specimen was not appreciably different from zero per cent; for these segments, the void ratio was taken as the average of the values determined for the colder segments. Finally, the initial degrees of saturation for all the specimens were computed on the basis of the overall average void ratio of the specimen.

A typical data sheet showing the various observed and computed quantities is shown on page 99.

Lastly, mention must be made of the fact that the average final degree of saturation of any one specimen (obtained by averaging the individual degrees of saturation of the various segments) was found to differ erratically from the initial degree of saturation of the same specimen. In view of the uncertainties involved in the calculation of the initial degree of saturation, and of the experimental errors in the computation of the final degree of saturation, such differences were deemed to be reasonable.



FIG. [1] TWO VIEWS OF THE ASSEMBLED APPARATUS

100

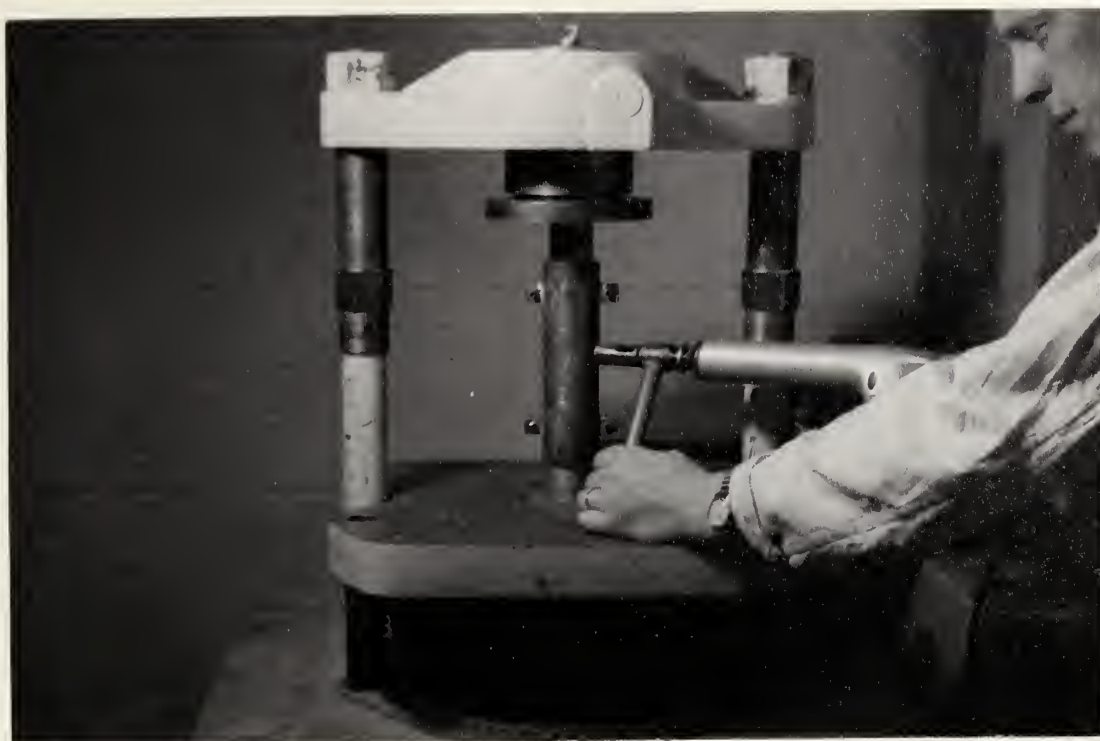


FIG. [2] COMPACTING THE SPECIMEN



FIG. [3] SPECIMENS READY TO BE MOUNTED





FIG. [4] LEVELLING THE SUPPORT
FOR THE SPECIMENS

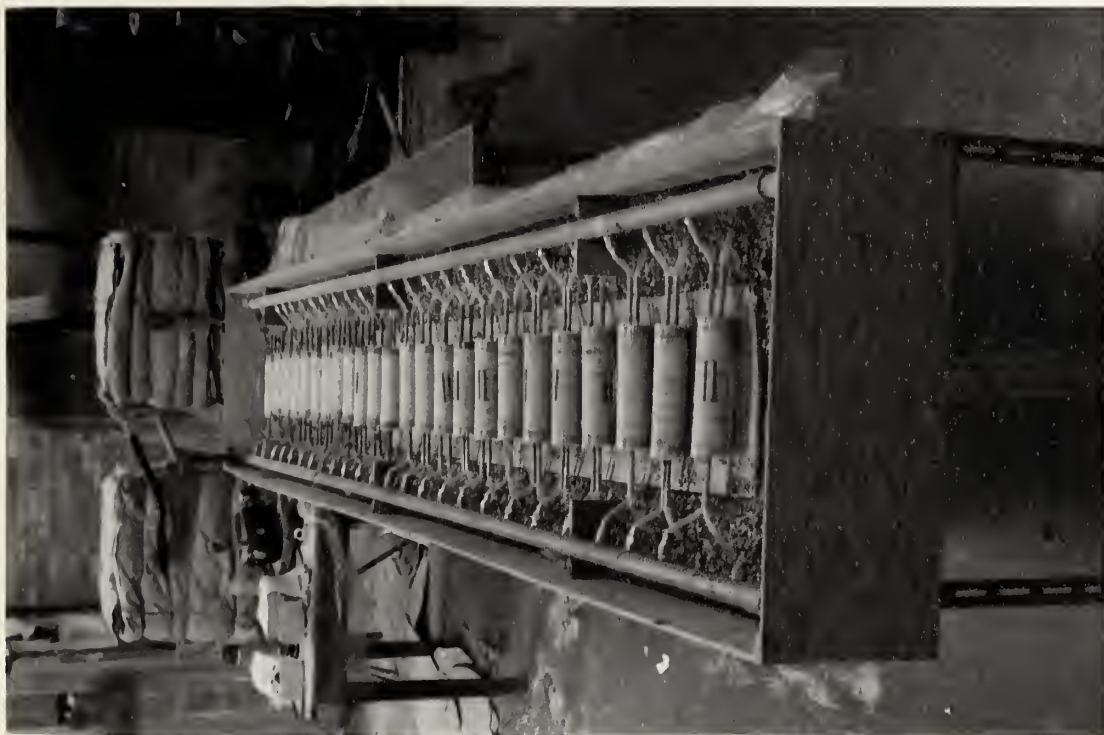


FIG. [5] SPECIMENS MOUNTED IN POSITION

100



FIG. [6] TAKING THERMO-COUPLE READINGS



FIG. [7] SPECIMEN MOULD AND ACCESSORIES





FIG. [8] ENLARGED VIEW OF SPECIMENS IN POSITION





FIG. [9] CLOSE-UP SHOWING THERMO-COUPLE LEADS

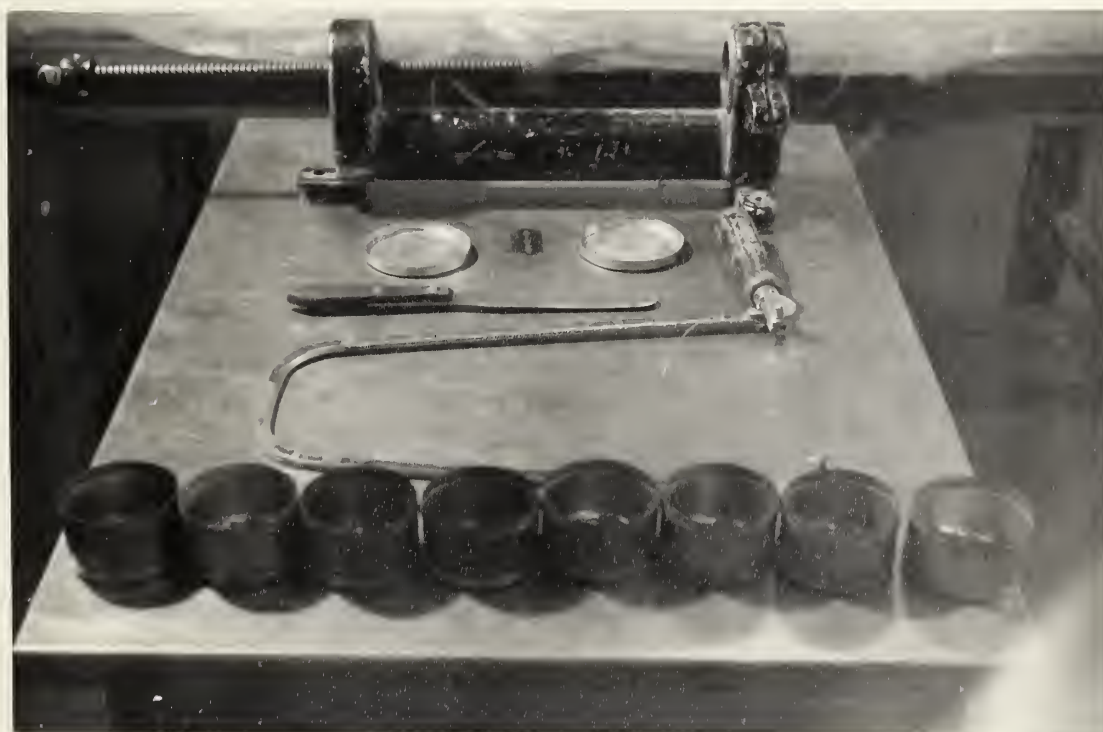


FIG. [10] ACCESSORIES USED IN SAMPLING OPERATIONS

CHAPTER VI

DISCUSSION OF RESULTS

6.1. Preliminary considerations:

Only one type of soil was used in the present series of tests; the compacted density, and hence the void ratio, of the specimens were constant within limits of experimental error. Although the initial density conditions of the various sections of the specimens were not determined, it is believed that the density conditions remained unaltered during thermo-osmotic flow.

The data obtained from the tests involve five variables as follows:

- i) the initial degree of saturation,
- ii) the final degree of saturation, or, the change in degree of saturation at various points along the specimen,
- iii) the distance along the specimen,
- iv) the elapsed time, and
- v) the temperature gradient.

As only two temperature gradients were employed, it is possible to eliminate the temperature gradient as a variable,

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by considering the two series separately. The remaining four variables have been combined in different ways to obtain four different sets of graphs for each series, each set emphasizing one aspect of the phenomenon. These sets of graphs will be taken up one at a time for a discussion of their implications.

It is well to point out that the initial degrees of saturation of the various specimens, originally prepared for the same moisture contents, were not exactly the same; the individual values were found to vary within ± 1 per cent (degree of saturation) from the arithmetic mean value for the group.

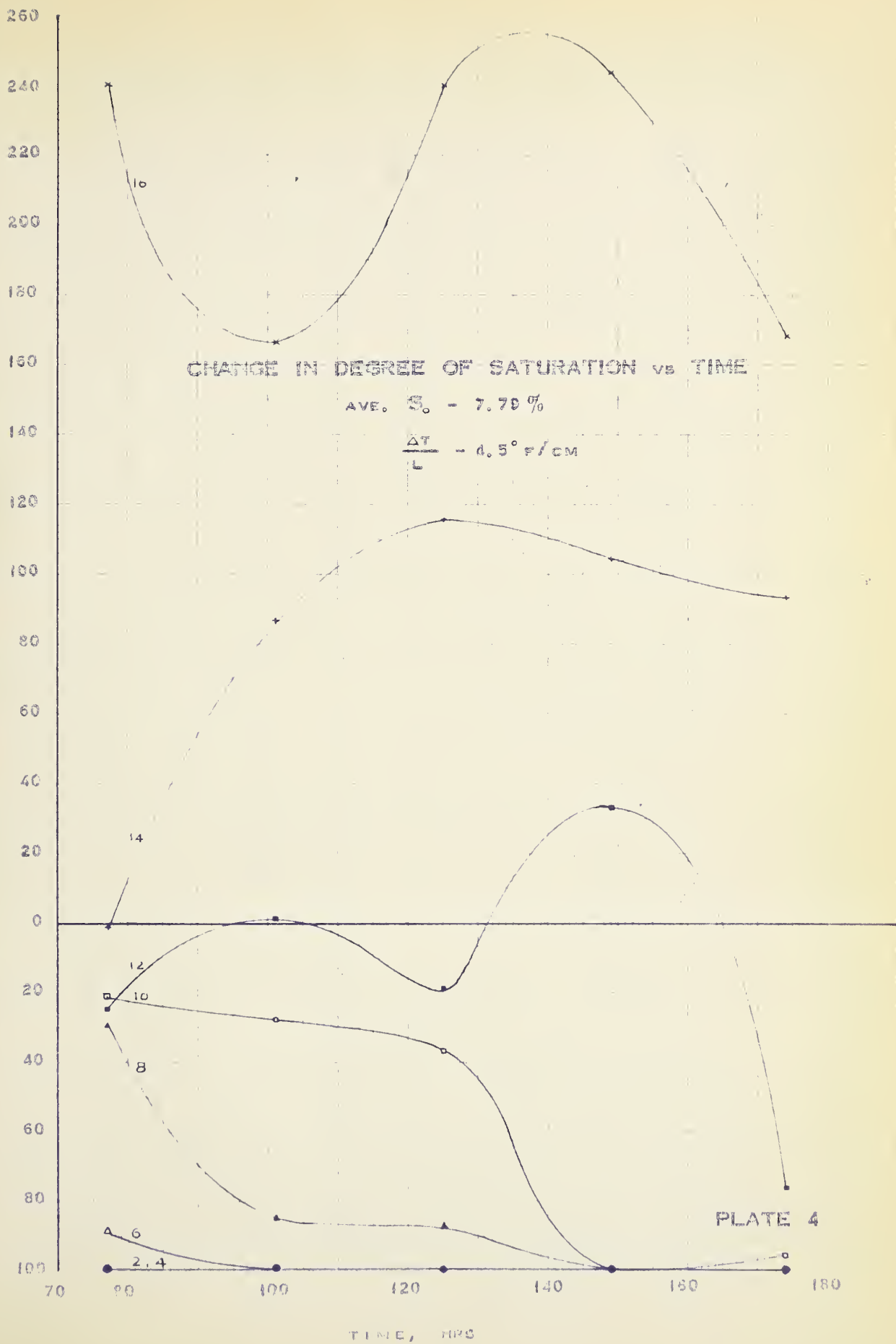
6.2. Change in degree of saturation vs. time:

The first set of graphs (Plates 4 to 15 inclusive) show how the degree of saturation of the different segments of the specimen varies with time; in order to eliminate the influence of minor differences in the initial degrees of saturation of the specimens having the same nominal initial degree of saturation, the change in degree of saturation has been expressed as a percentage of the initial degree of saturation; each curve represents the changes taking place at the noted distance from the hot end.

6.2.1. For the temperature gradient of 4.5°F./cm. :

Graphs 4 to 9 (pp. 108-113) are for specimens subjected to the high temperature gradient of 4.5°F./cm. In the driest group of specimens, the degree of saturation for the first

PER CENT. CHANGE IN DEGREE OF SATURATION



CHANGE IN DEGREE OF SATURATION vs TIME

AVE. $S_o = 16.93\%$

$$\frac{\Delta T}{L} = 4.5^\circ F/CM$$

PER CENT. CHANGE IN DEGREE OF SATURATION

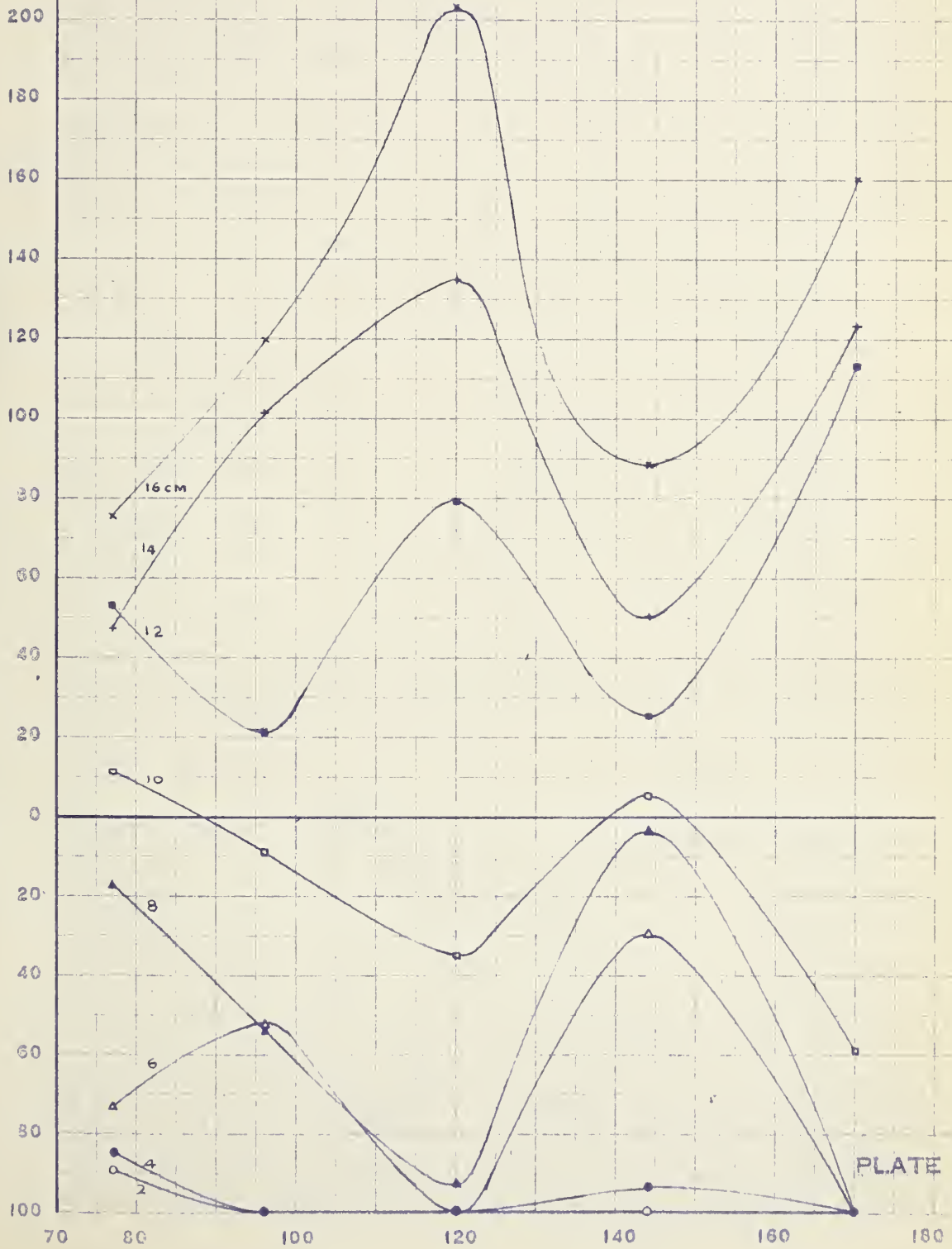


PLATE 5

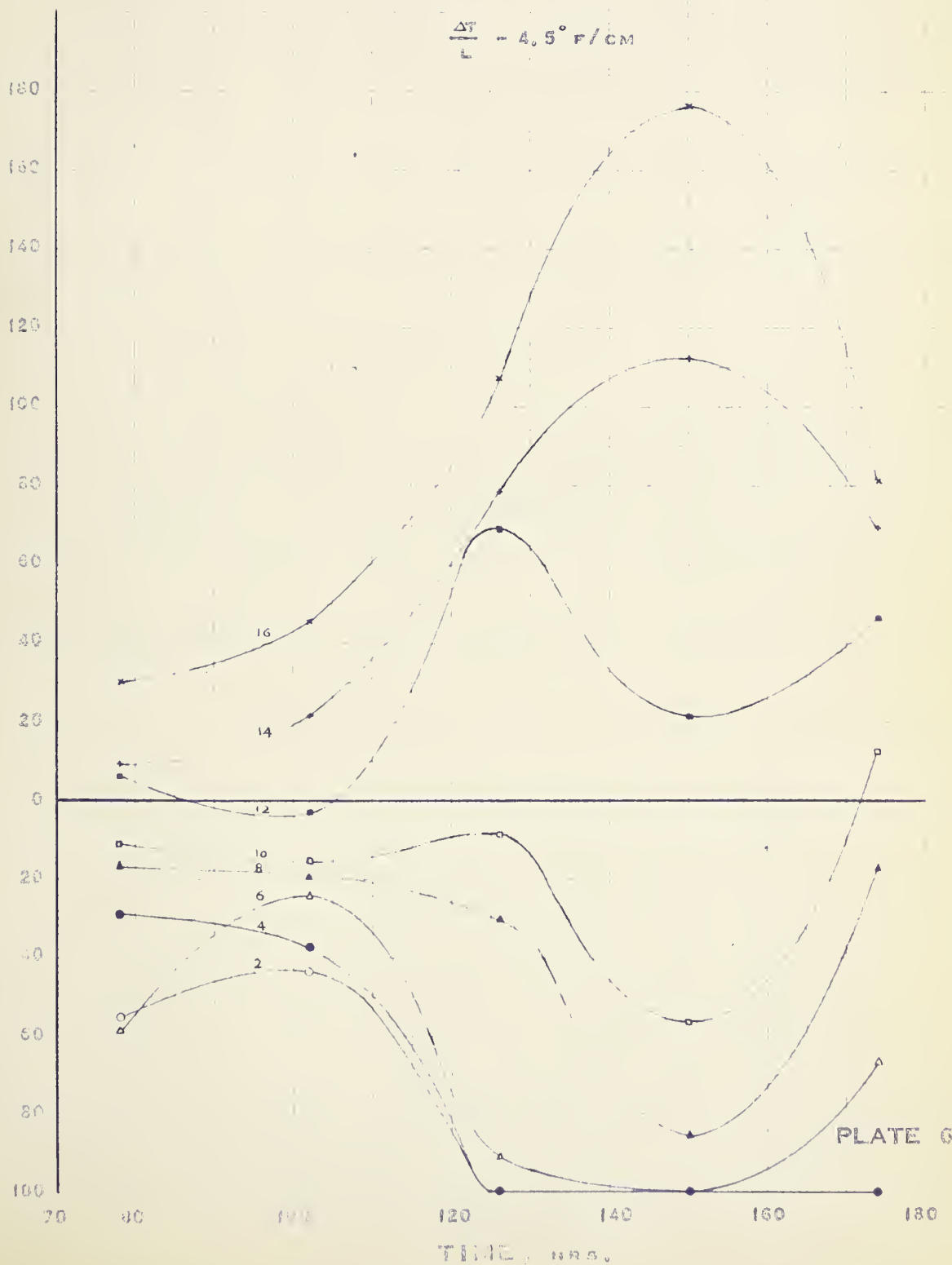
TIME, HRS.

CHANGE IN DEGREE OF SATURATION vs TIME

AVE. $S_o = 13.71\%$

$$\frac{\Delta T}{L} = 4.5^\circ \text{ F/CM}$$

PER CENT. CHANGE IN DEGREE OF SATURATION



CHANGE IN DEGREE OF SATURATION vs TIME

AVE. $S_o = 23.77\%$

$$\frac{\Delta T}{L} = 4.5^\circ/\text{cm}$$

PER CENT. CHANGE IN DEGREE OF SATURATION

80
60
40
20
0
20
40
60
80
100

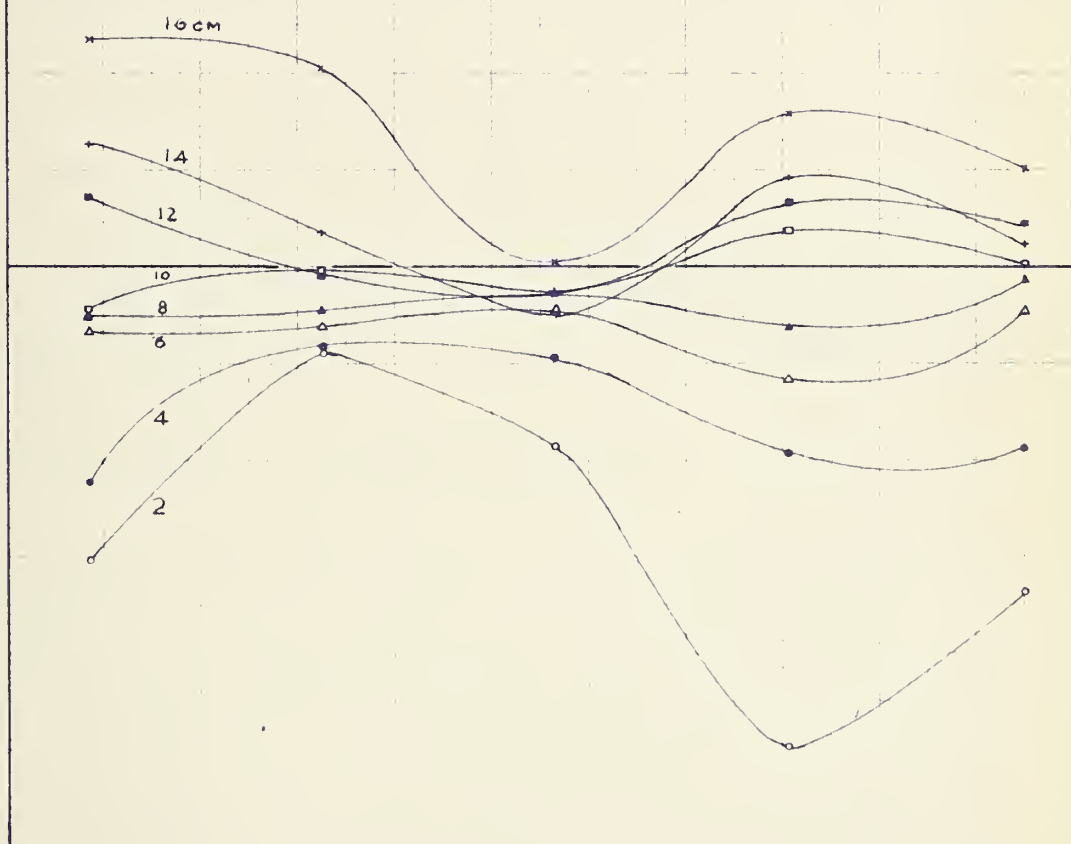


PLATE 7

70 80 100 120 140 160

TIME, HRS.

CHANGE IN DEGREE OF SATURATION vs TIME

AVE. $S_o = 30.91\%$

$$\frac{\Delta T}{L} = 4.5^\circ\text{F}/\text{CM}$$

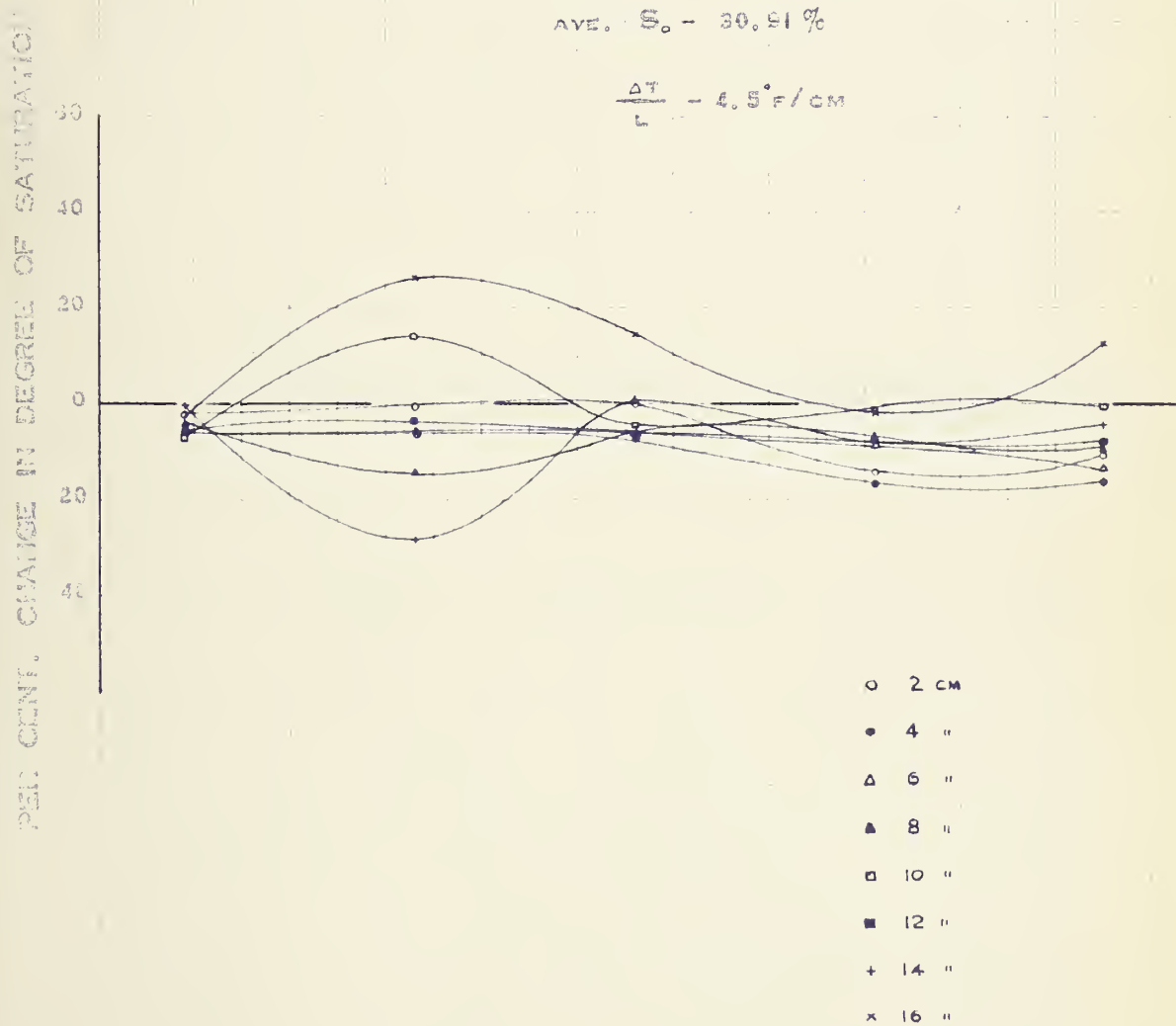


PLATE 8

70

90

100

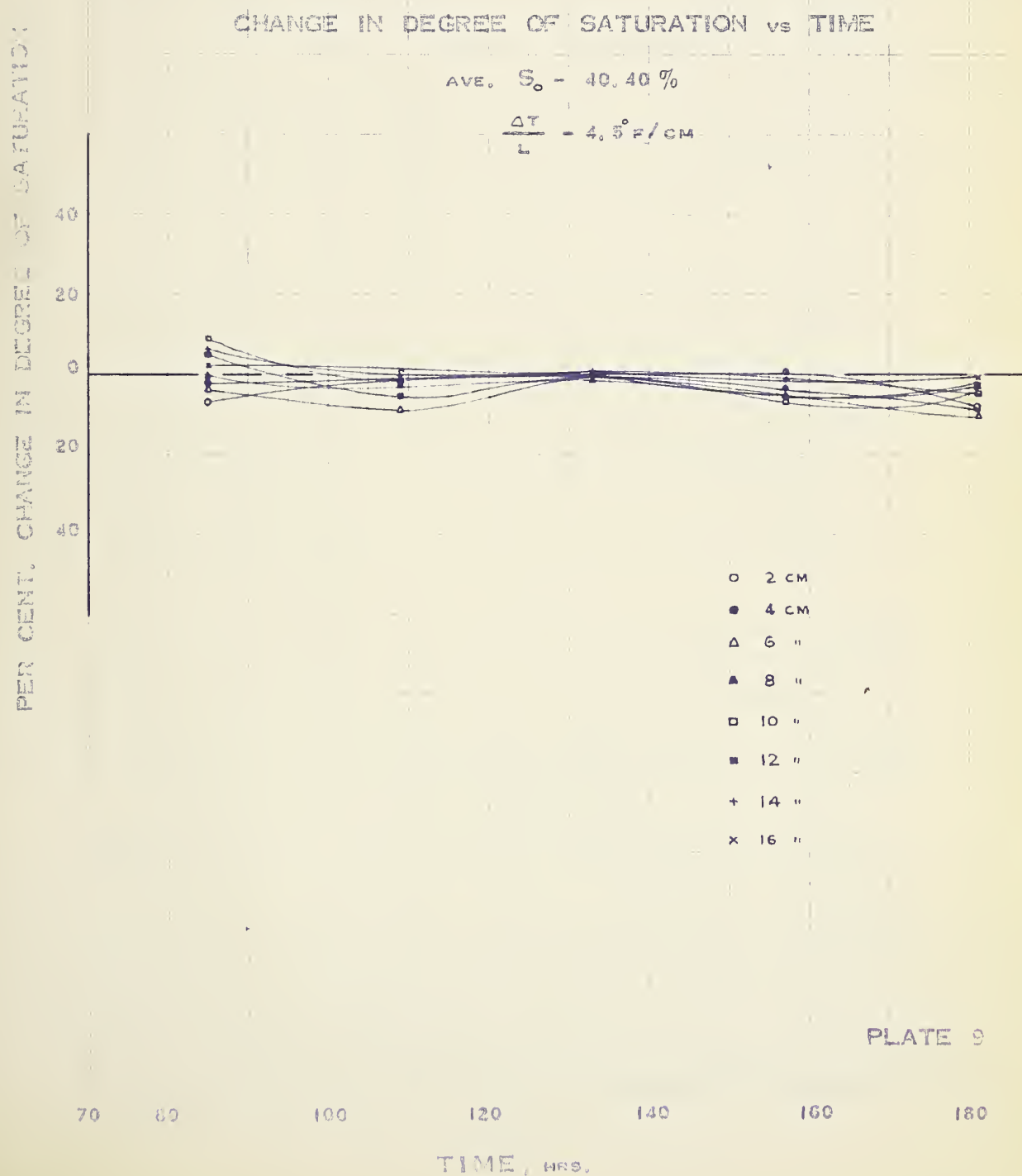
120

140

160

180

TIME, HRS.



4 cm. section decreases to practically zero, and does not vary appreciably with time. For the next three 2 cm. sections of the specimen, the degree of saturation decreases with time. At the 10 cm. section, the degree of saturation reaches nearly zero at 149 hours, but undergoes a small increase in the next 24 hours. In contrast, the degrees of saturation of the soil in the last three 2 cm. sections appear to fluctuate considerably with time. Such fluctuations indicate that the water accumulated at the cold end must be flowing backwards under the influence of some potential gradient. The absence of any such fluctuations at the warmer sections of the specimen, taken in conjunction with the observed fact that these sections are practically dry, suggest that the return flow is in the liquid phase, and that the flow takes place up to such distances behind the cold end, as permitted by the presence of continuous water films.

Ample evidence of the fluctuations in the degree of saturation of the various sections of the specimens is also provided by the remaining five graphs of the set. It will be noticed that, with increasing degree of initial saturation, the zone of drying^{*} moves towards the hot end progressively, till it disappears altogether with relatively wet specimens (degree of saturation greater

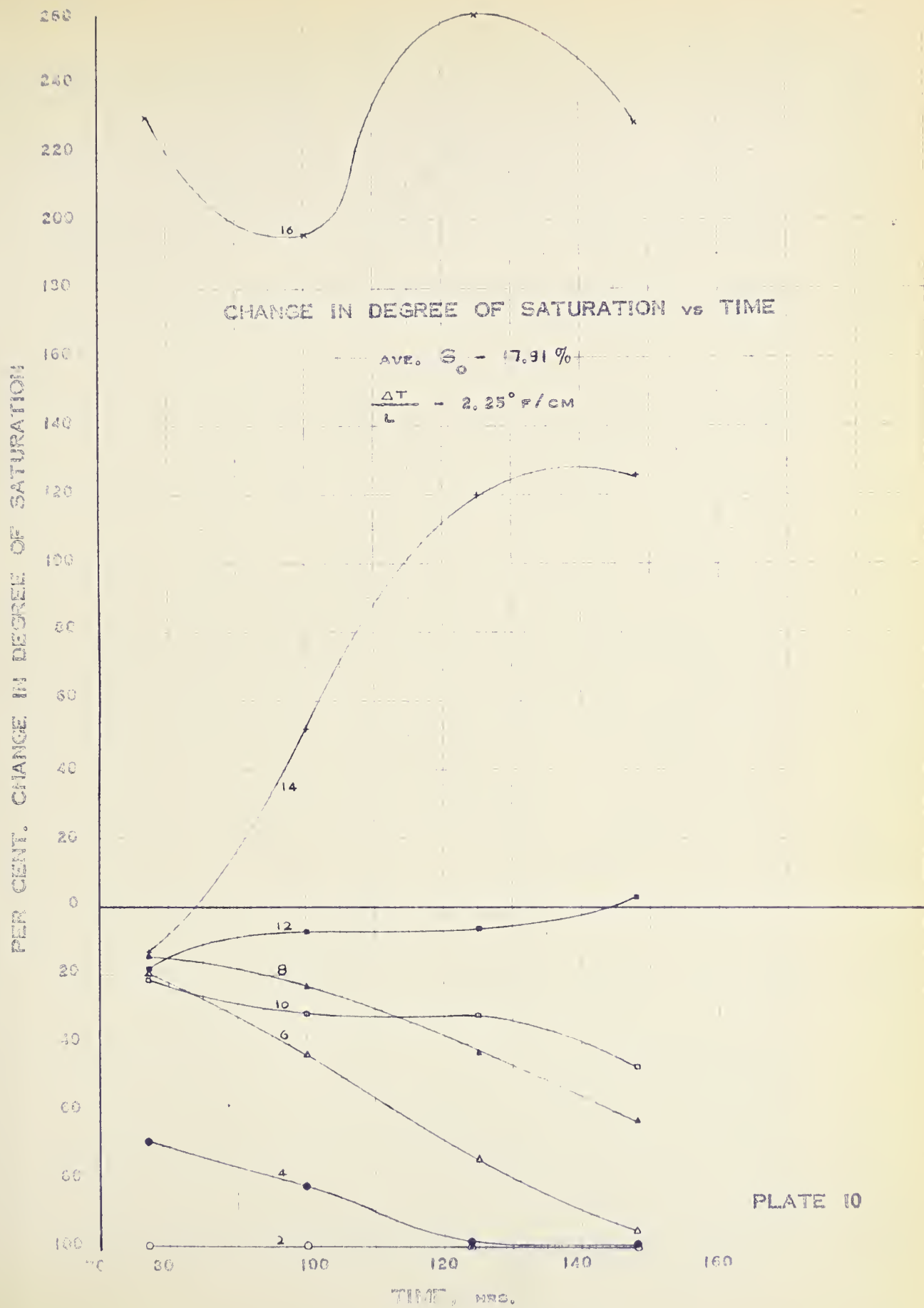
* The "zone of drying" refers to that section of the specimen where the decrease in degree of saturation is nearly 100 per cent.

than about 20%: this is illustrated by Plates 25-31 also). It will also be noticed that, as the initial degree of saturation increases, larger and larger portions of the specimen undergo fluctuations in the change in degree of saturation. This is to be expected, since, with increasing degree of wetness of the specimen, continuous conducting water films are present for longer distances back of the cold face and so the return liquid flow extends to greater and greater distances from the cold end.

A final point worthy of consideration is that the accumulation at the cold end is the greatest for the driest specimen, on a percentage basis. However, in terms of actual magnitudes, this is really not so; comparing the specimens of the first and second groups (initial saturation 7.8% and 16.9% respectively), it is easily checked that an increase in the degree of saturation of about 260% in the former case is numerically smaller than a similar increase of about 200% in the latter.

6.2.2. For a temperature gradient of $2.25^{\circ}\text{F./cm.}$ (Plates 10-15)

The general trends in all cases of this series are well defined. The same general conclusions deduced from the first series are equally applicable to this series, thus supporting the validity of the conclusions. The main difference between the two sets of plots lies in the extent of drying zones of parallel specimens and the corollary extent of the region of recirculation of water. This difference



PER CENT. CHANGE IN DEGREE OF SATURATION

CHANGE IN DEGREE OF SATURATION vs TIME

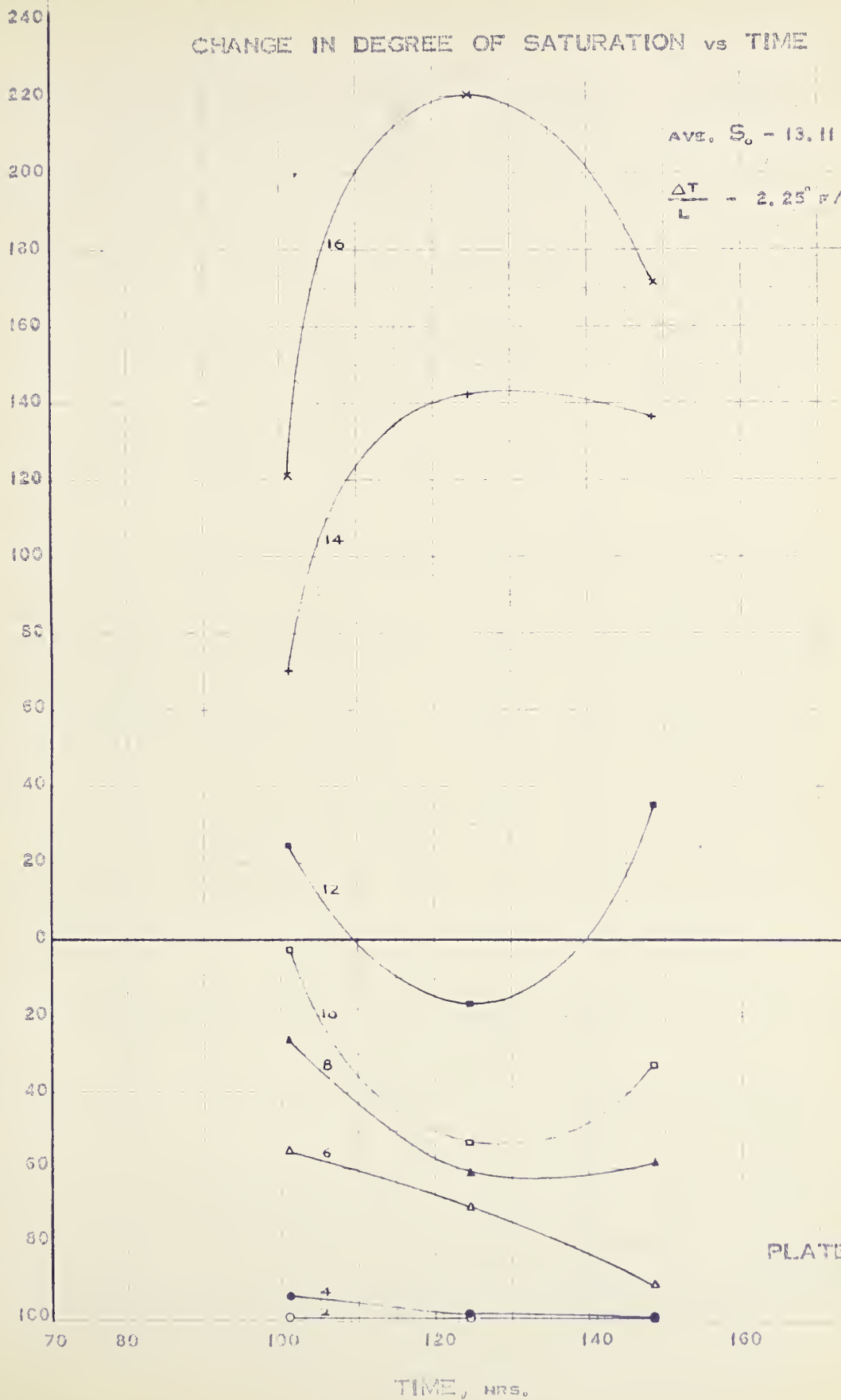
AVE. $S_o = 13.11\%$ $\frac{\Delta T}{L} = 2.25^\circ \text{ W/CM}$ 

PLATE II

CHANGE IN DEGREE OF SATURATION vs TIME

PER CENT. CHANGE IN DEGREE OF SATURATION

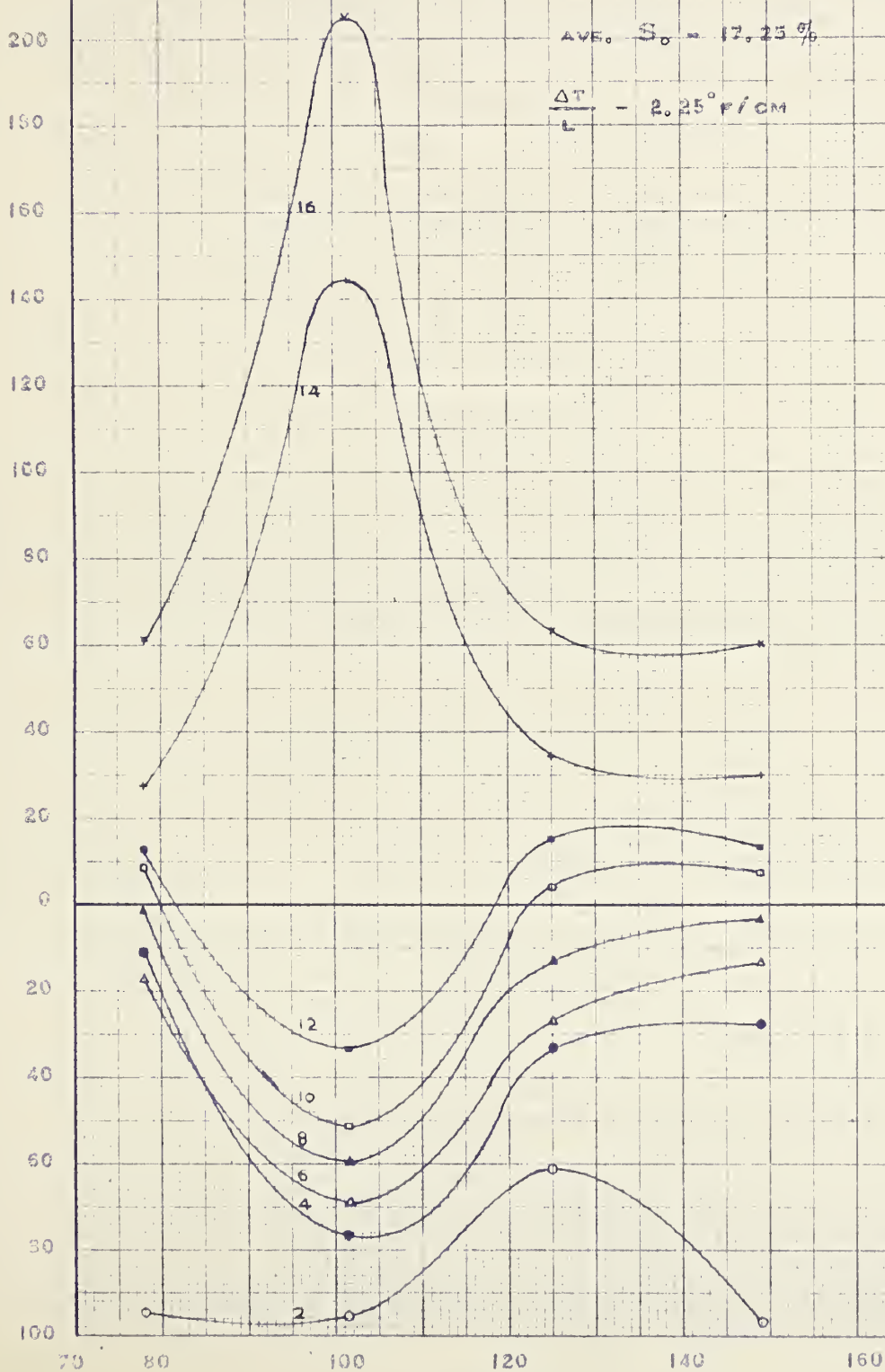
AVE. $S_o = 17.25\%$ $\frac{\Delta T}{L} = 2.25^\circ \text{F}/\text{CM}$ 

PLATE 12

PER CENT. CHANGE IN DEGREE OF SATURATION

CHANGE IN DEGREE OF SATURATION vs TIME

AVE. $S_o = 28.27\%$

$$\frac{\Delta T}{L} = 2.25^\circ \text{F}/\text{CM}$$

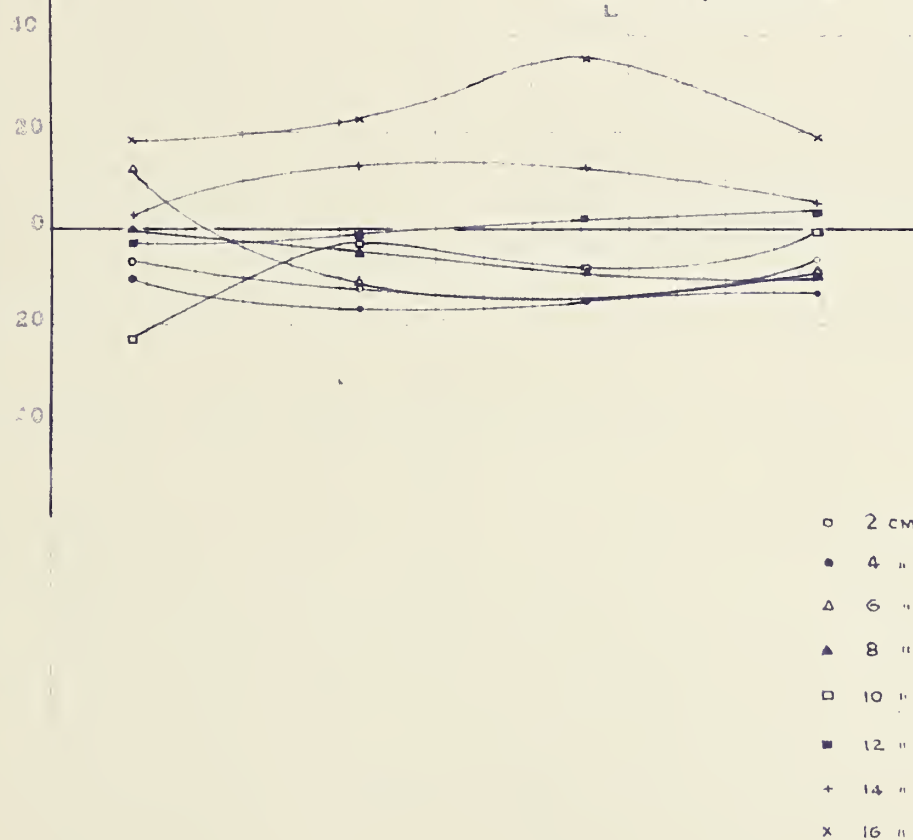


PLATE 13

80

100

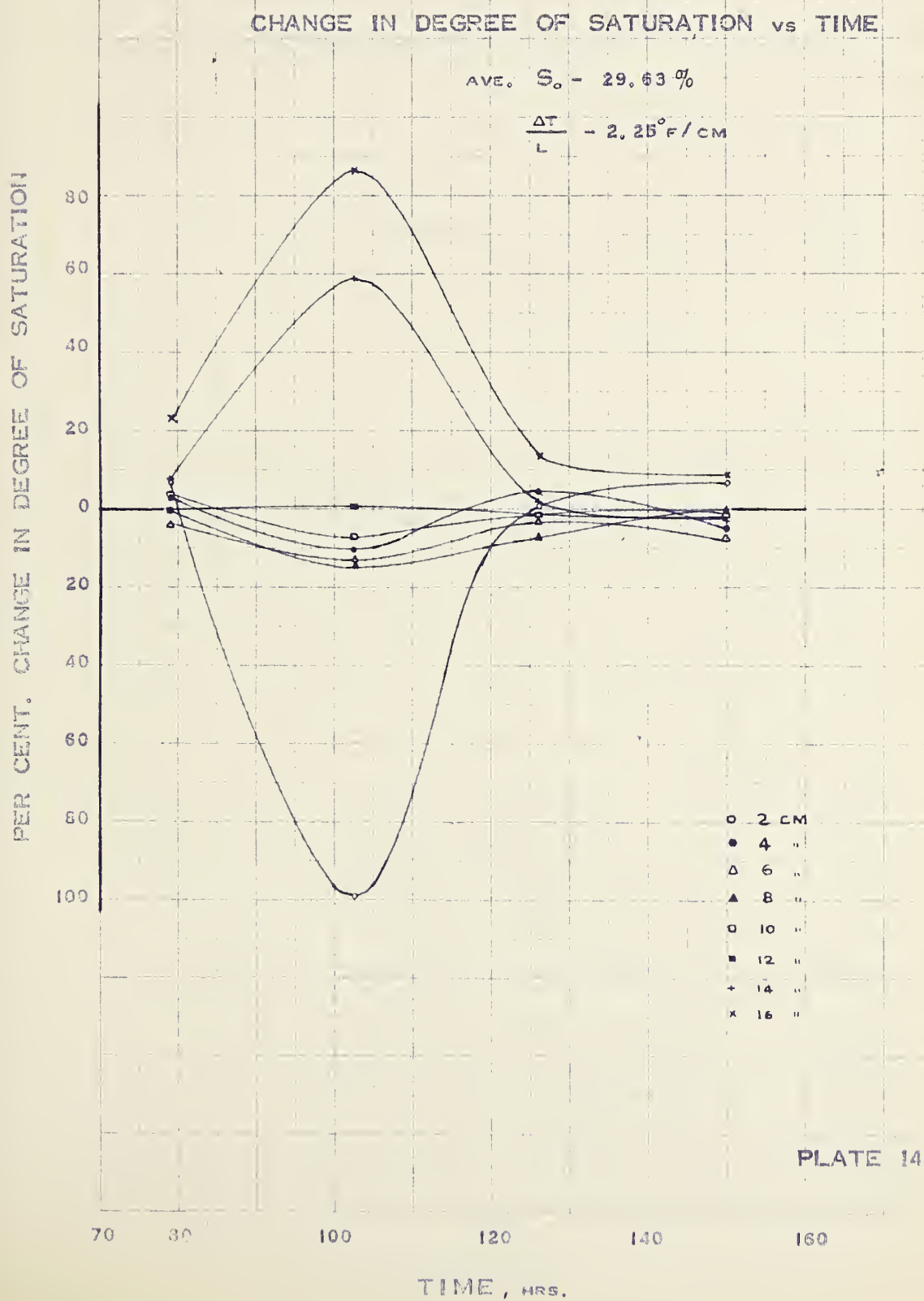
120

140

160

180

TIME, HRS.



PER CENT. CHANGE IN DEGREE OF SATURATION

CHANGE IN DEGREE OF SATURATION vs TIME

AVE. $S_o = 37.05\%$

$$\frac{\Delta T}{L} = 2.25^\circ \text{F}/\text{CM}$$

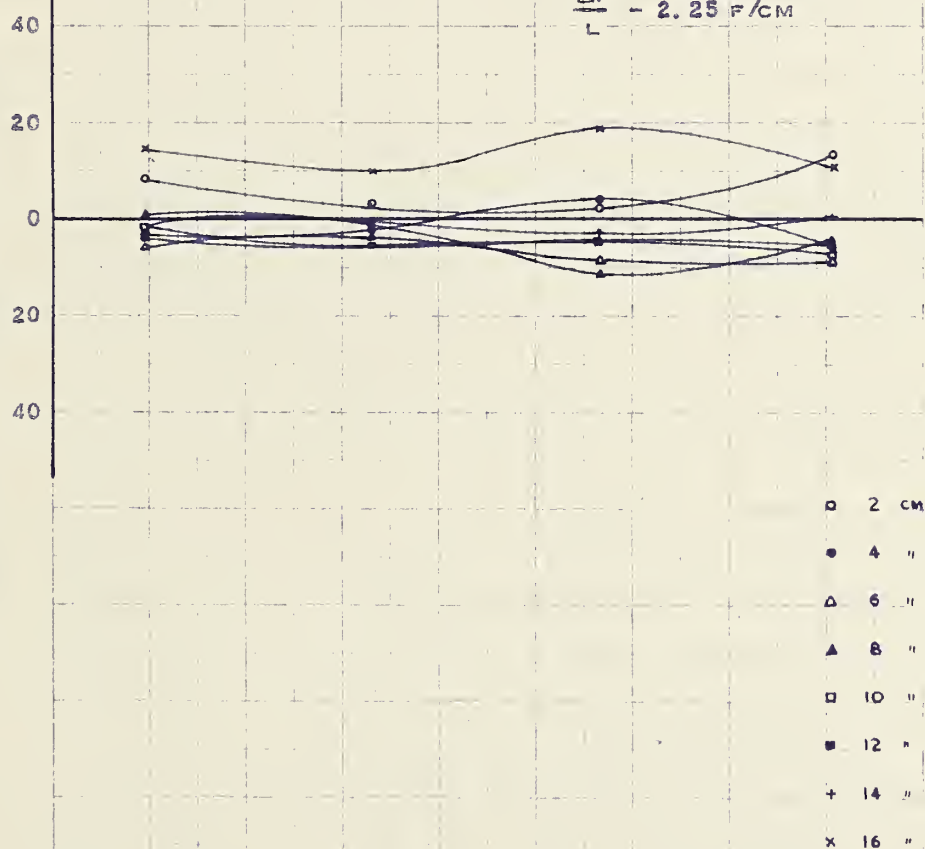


PLATE 15

70 80 100 120 140 160

TIME, HRS.

arises from the fact that the vapour pressure gradients were smaller in the latter case than in the former; but the initial relative humidity and all other conditions were nearly identical. Hence, the rate of building up of a suction gradient from cold to the hot side was greater in the latter case than in the former, thus starting a return flow before appreciable portions at the warmer end could become so dry as to prevent active liquid flow. Thus, the data support Hutcheon's hypothesis (see Section 4.3.4.).

An inexplicable situation presents itself in the case of the group of specimens with an initial degree of saturation of 29.6% (Plate 14). The changes in the degrees of saturation of the various specimens of the group except one are very small, within ± 15 per cent of the initial value. The second specimen (taken out after 102.5 hours) shows a reduction of 99.1 per cent at the warmest end and an equal accumulation at the 14 cm. section immediately back of the cold end. As the graphs in Plate 13 show, the maximum net transfer in the group with 23.3 per cent saturation is only about 35 per cent, and at higher degrees of saturation, one should expect the transfer to be smaller still. This is indeed the case with the last group of specimens with 37.1 per cent saturation (Plate 15). This anomalous situation is further discussed in a subsequent section.

6.3. Change in degree of saturation vs. initial saturation:

The same data have been plotted in a different manner, wherein the per cent change in the degree of saturation has been plotted as a function of the initial saturation for various distances along the specimen (Plates 16-24 inclusive). For the sake of clarity, the elapsed time has been treated as constant for any one series of graphs of the set, although all the specimens of any particular series were not sampled simultaneously. The effect of initial saturation on the net change is brought out very clearly by this series of graphs.

6.3.1. For the higher temperature gradient:

The most striking feature of all the graphs (16-20) is that the per cent change decreases rapidly with increasing degree of initial saturation, and that the family of curves in any one group seems to converge at a particular value of the initial saturation. The point of convergence is not the same for the various time intervals but ranges between 30 and 40 per cent initial degree of saturation. This is explainable as follows: the moisture profile at any time is governed by the rate of vapour diffusion from the hot to the cold side as well as by the rate of recirculation from the cold to the hot side; these rates are different for specimens with different degrees of initial saturation, as both the rates are ultimately governed by the relative humidity of the soil water, the amount of air filled pores

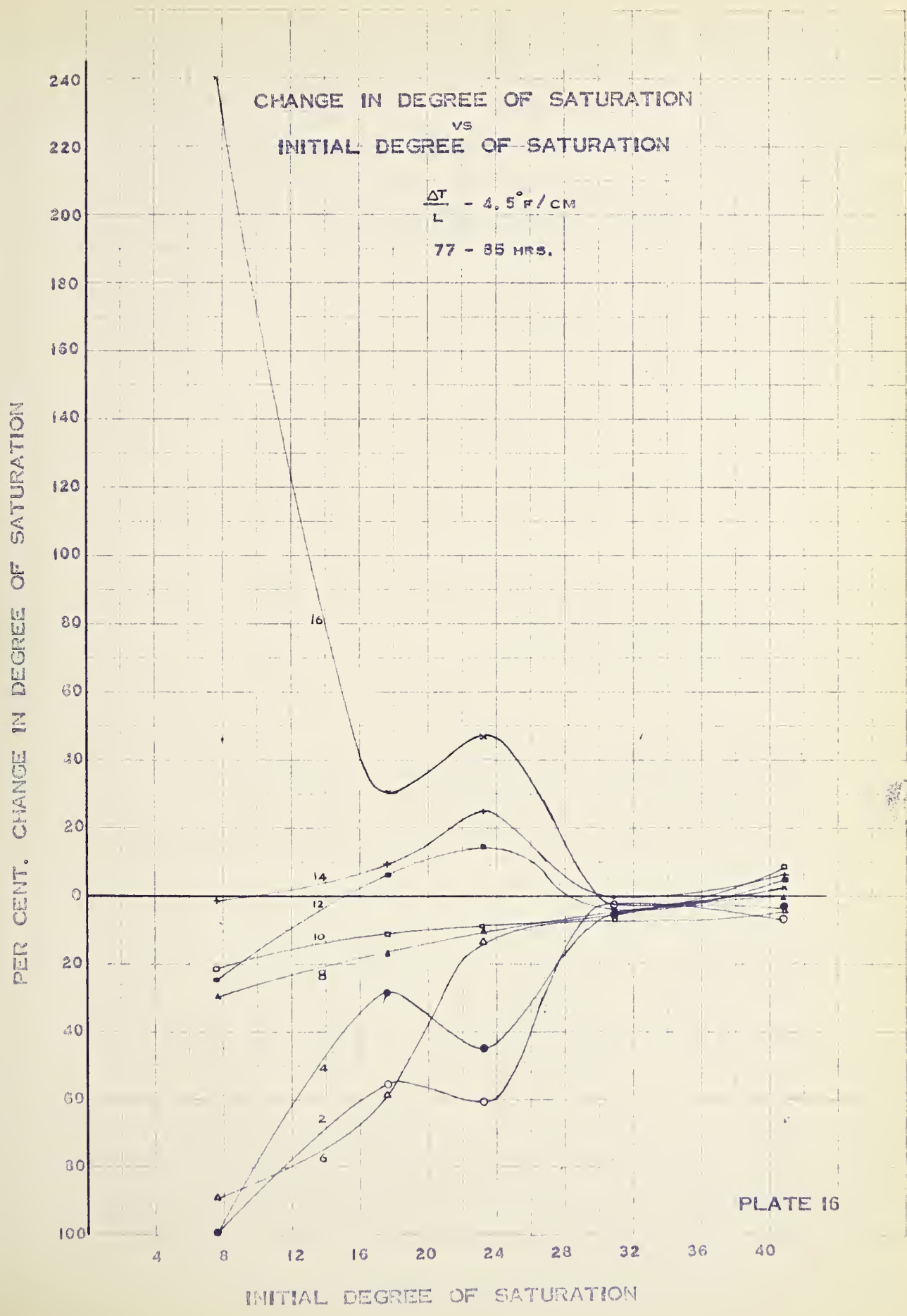


PLATE 16

CHANGE IN DEGREE OF SATURATION

VS

INITIAL DEGREE OF SATURATION

$$\frac{\Delta T}{L} = 4.5^\circ \text{ F/CM}$$

101 - 109 HRS.

PER CENT. CHANGE IN DEGREE OF SATURATION

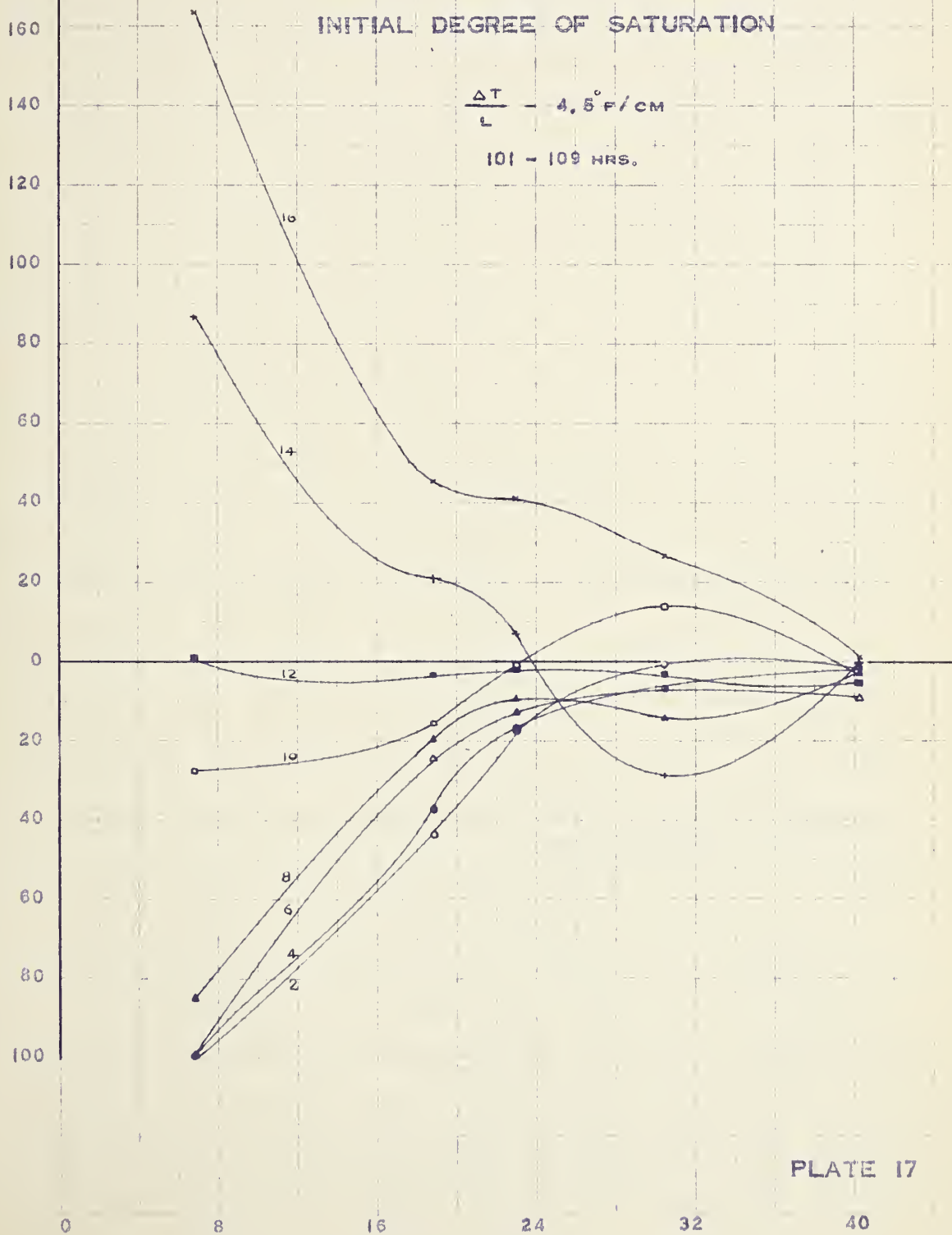
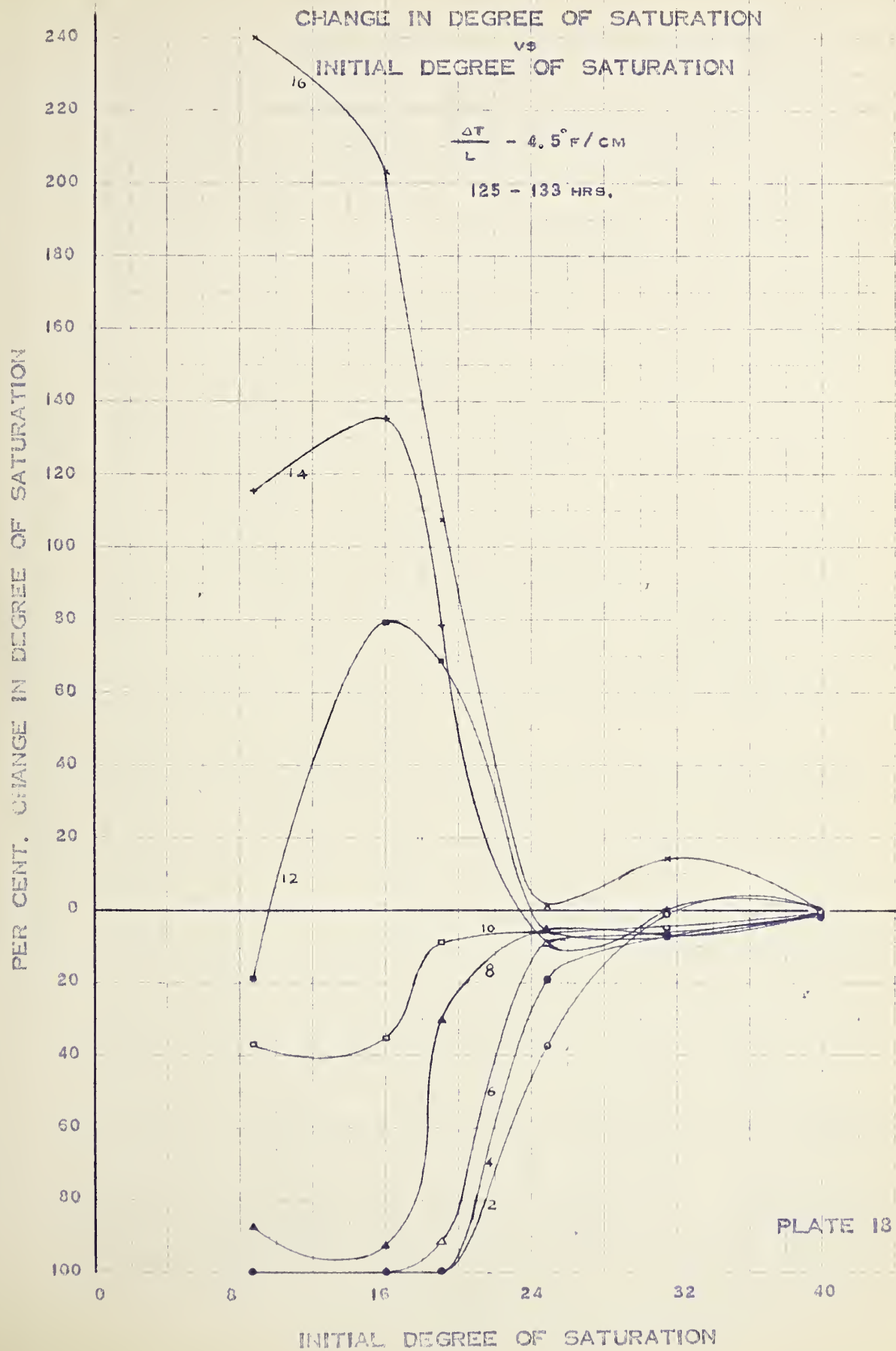
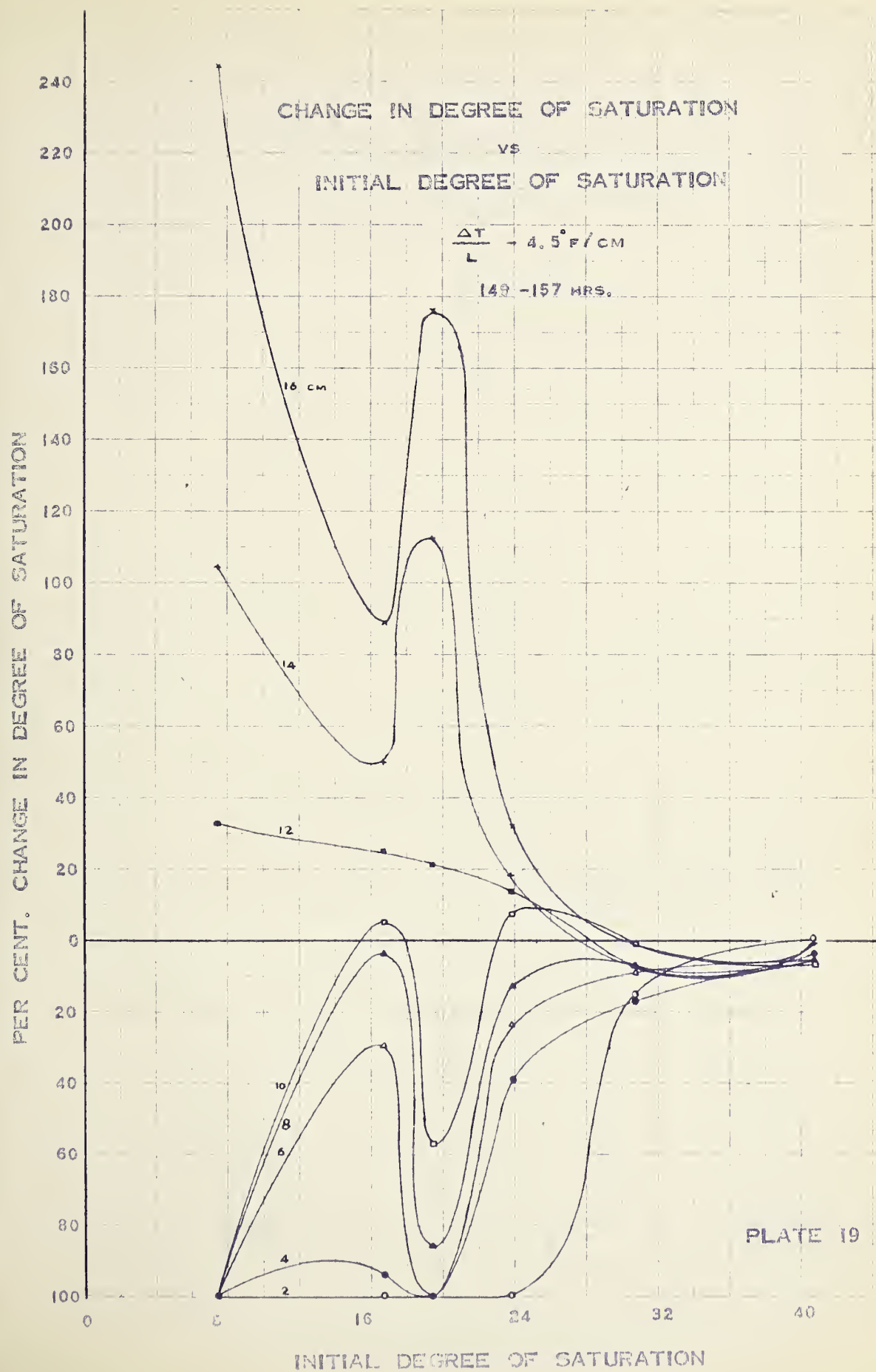


PLATE 17

INITIAL DEGREE OF SATURATION





CHANGE IN DEGREE OF SATURATION

vs.

INITIAL DEGREE OF SATURATION

PER CENT. CHANGE IN DEGREE OF SATURATION

$$\frac{\Delta T}{L} = 4.5^\circ \text{F/CM}$$

174 - 181 HRS.

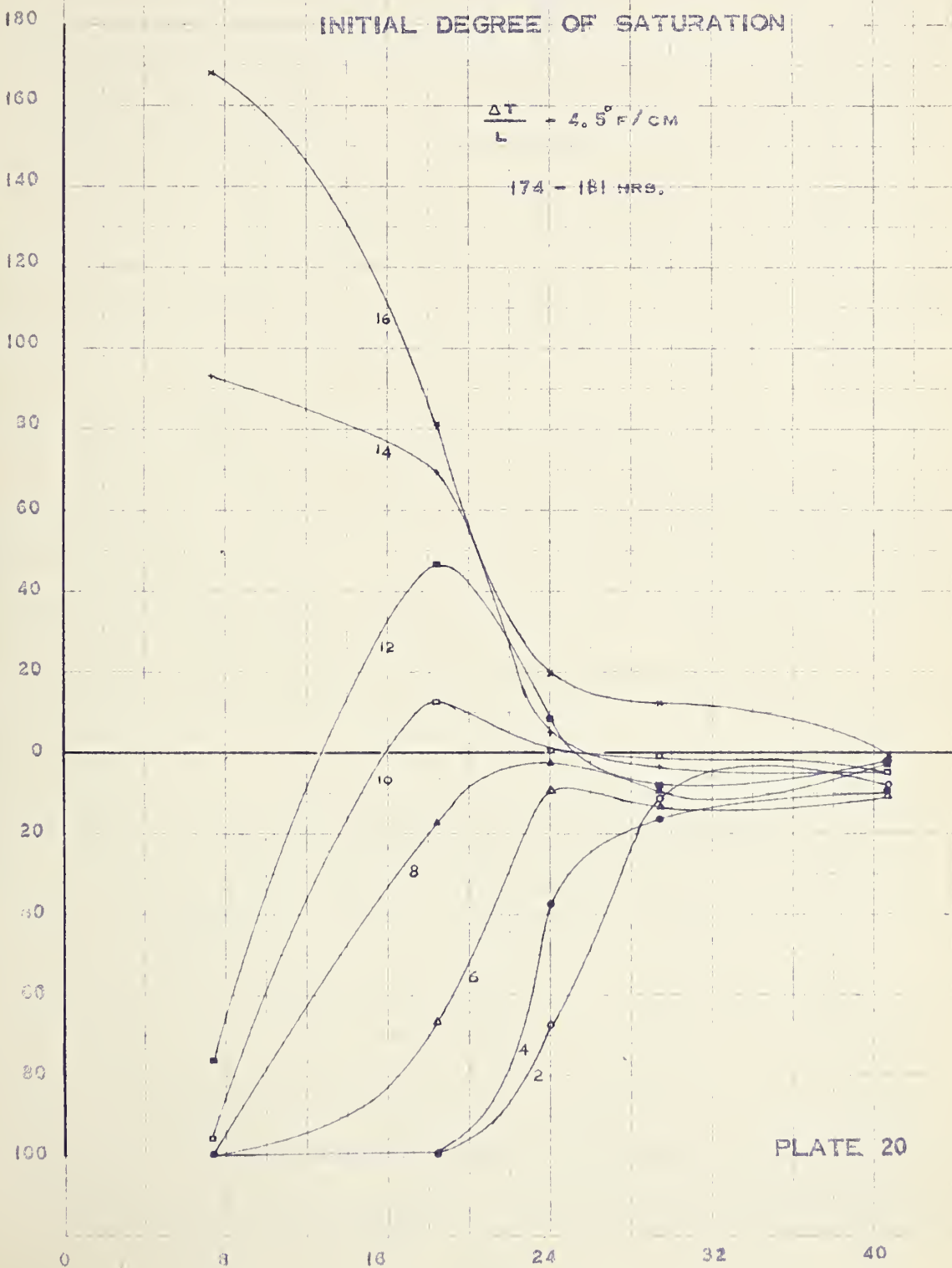


PLATE 20

INITIAL DEGREE OF SATURATION

CHANGE IN DEGREE OF SATURATION
vs
INITIAL DEGREE OF SATURATION

$$\frac{\Delta T}{L} = 2.25^\circ \text{F/CM}$$

77 - 79.5 MRS.

CHANGE IN DEGREE OF SATURATION

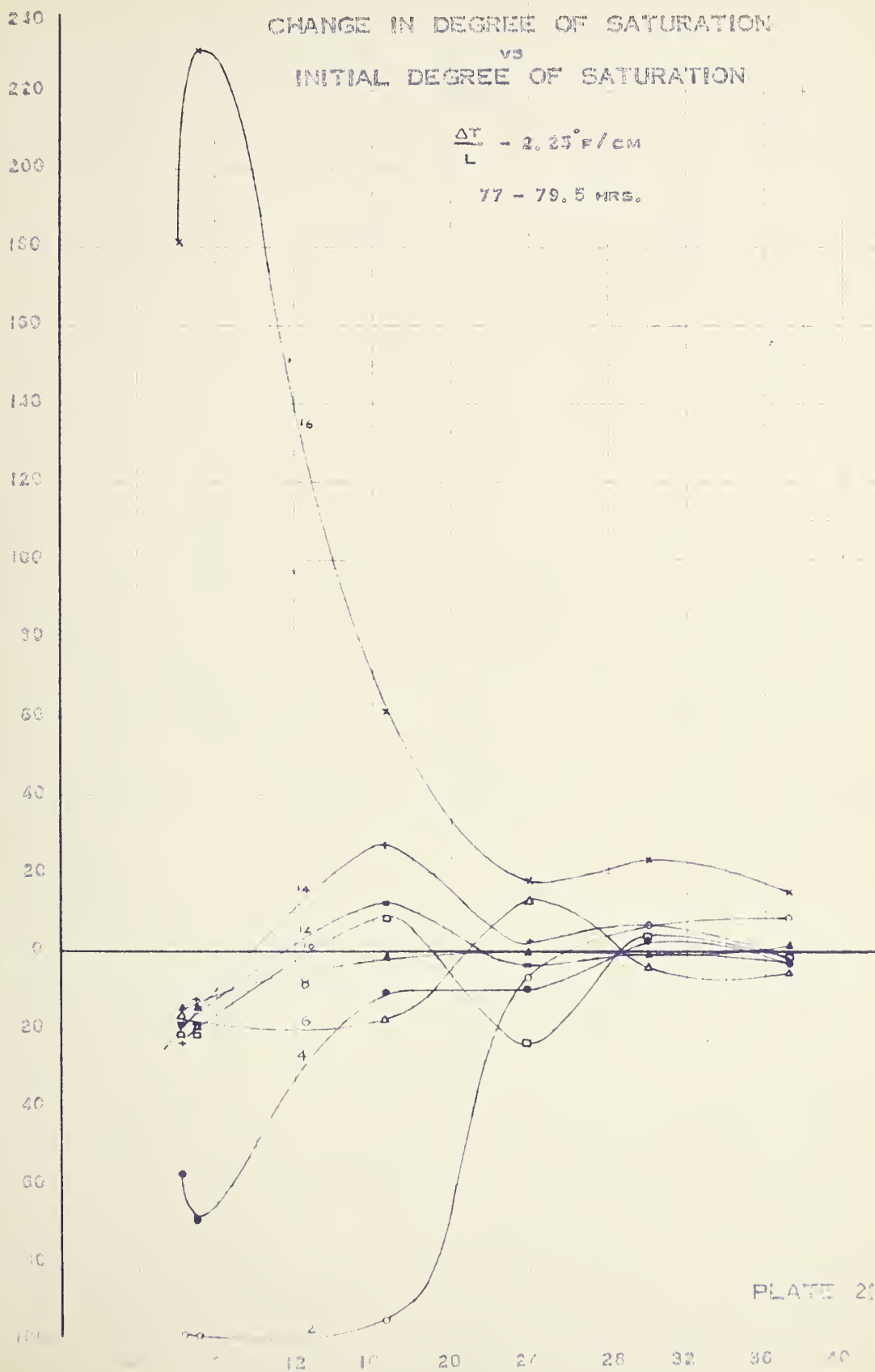


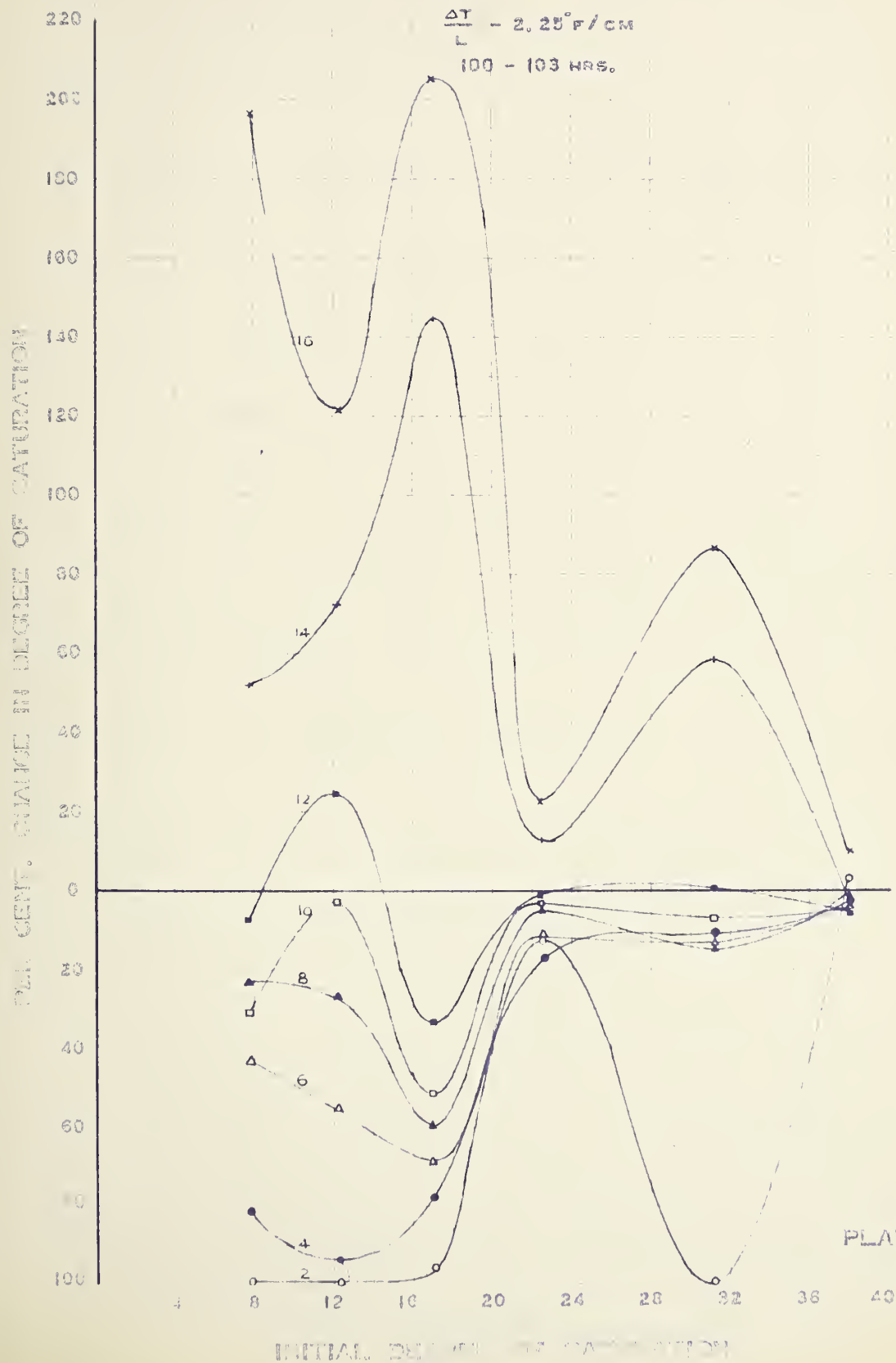
PLATE 21

INITIAL DEGREE OF SATURATION

CHANGE IN DEGREE OF SATURATION
VS
INITIAL DEGREE OF SATURATION

$$\frac{\Delta T}{L} = 2.25^\circ \text{F}/\text{cm}$$

100 - 103 HRS.



CHANGE IN DEGREE OF SATURATION
VS
INITIAL DEGREE OF SATURATION

$$\frac{\Delta T}{L} = 2.25^\circ \text{F}/\text{CM}$$

124 - 125.5 HRS.

CHANGE IN DEGREE OF SATURATION

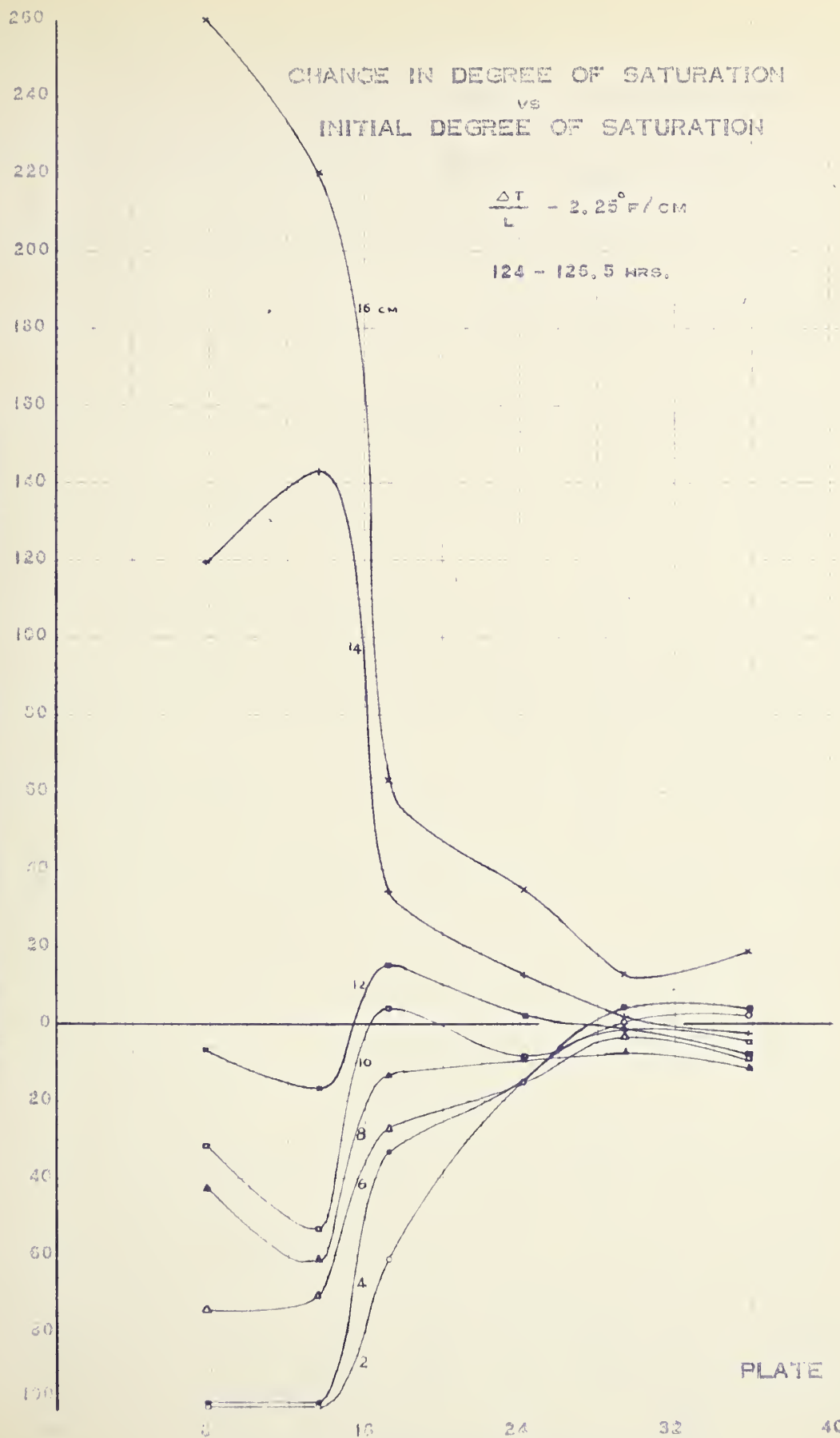
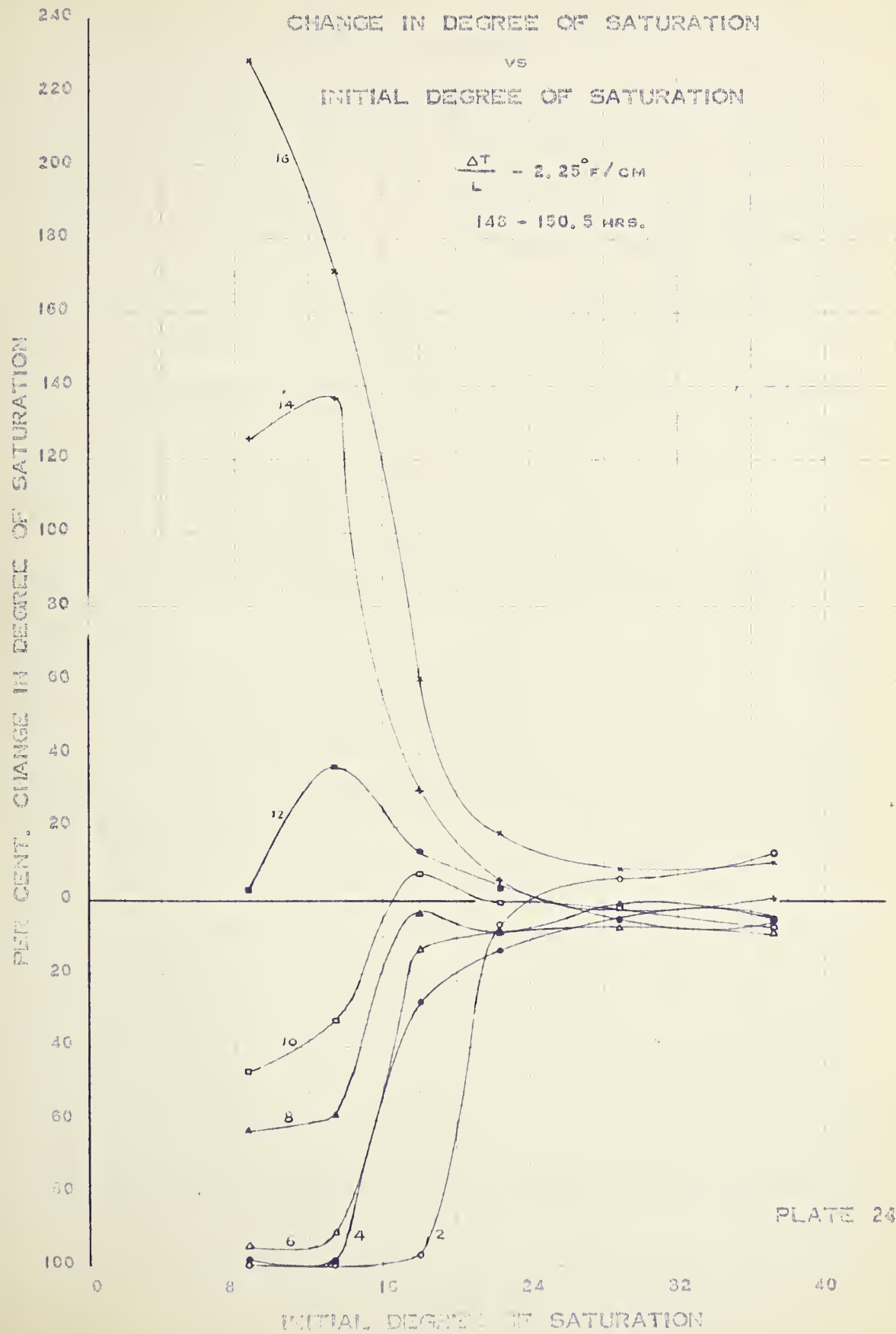


PLATE 23

INITIAL DEGREE OF SATURATION



and the unsaturated permeability, all of which are in some manner related to the degree of saturation. Thus, when two specimens with different degrees of saturation are taken out after a certain elapsed time, vapour diffusion may be dominant in one case, while recirculation may be dominant in the other. The same sequence of events is not likely to be maintained at a different elapsed time, except probably when both specimens have independently attained a state of dynamic equilibrium. Hence, the point of convergence cannot become stationary unless all specimens have attained dynamic equilibria, when the individual moisture profiles become independent of the elapsed time.

The preceding argument also serves to explain the undulating nature of the individual members of each family of curves.

6.3.2. For the smaller temperature gradient:

These families of curves (21-24) need no special comment, since the general trends are exactly similar to the previous ones.

6.4. Moisture profiles at various times:

In plates 25 to 36 inclusive, the final degrees of saturation at various distances along the individual specimens have been plotted as a function of the distance from the hot end. In order to bring out the variation of the moisture profile with time, the data obtained from various specimens having approximately the same initial degree of

MOISTURE PROFILES AT VARIOUS TIMES

AVE. $G_o = 7.79\%$

$$\frac{\Delta T}{L} = 4.5^\circ F/cm$$

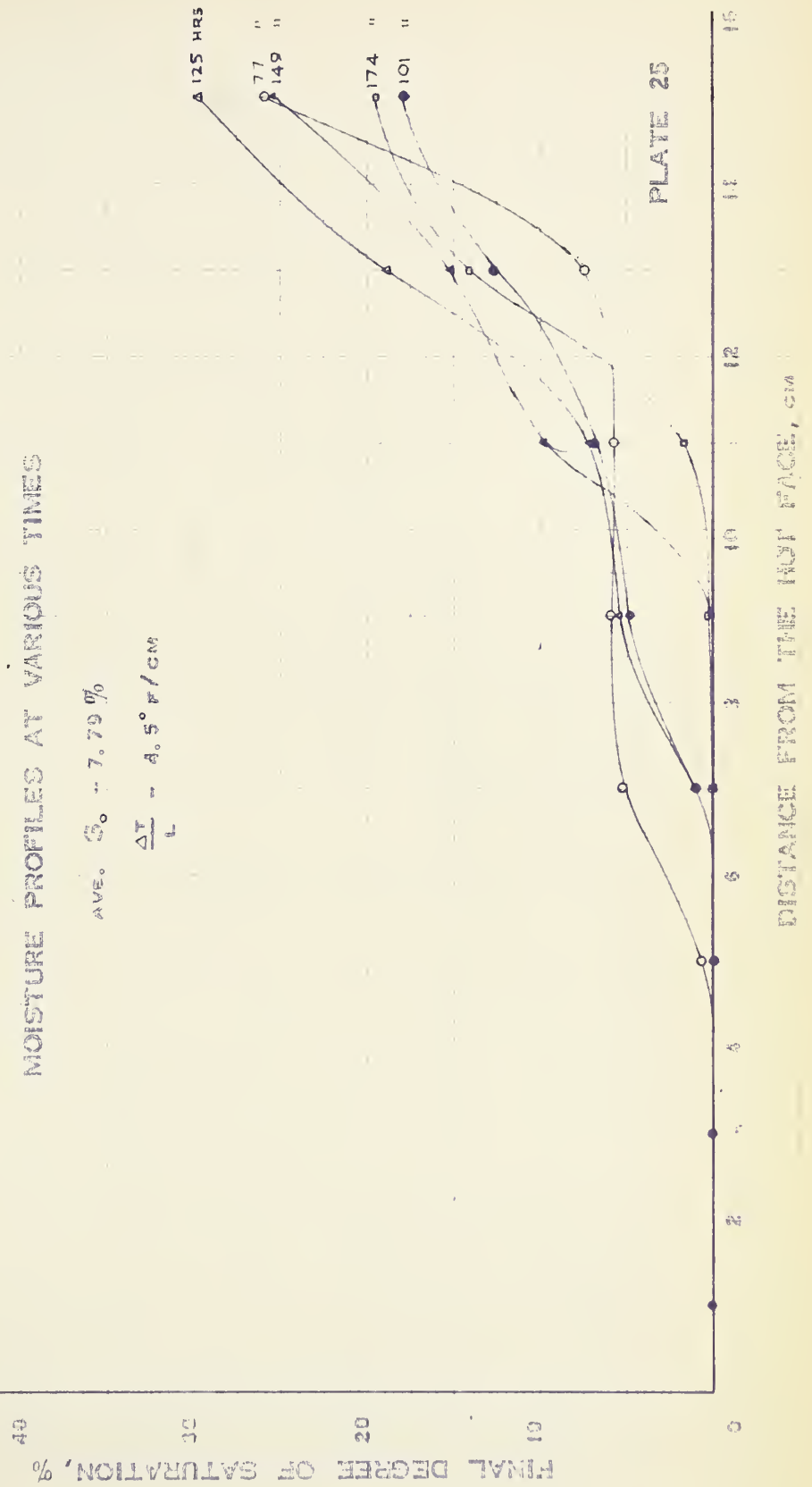
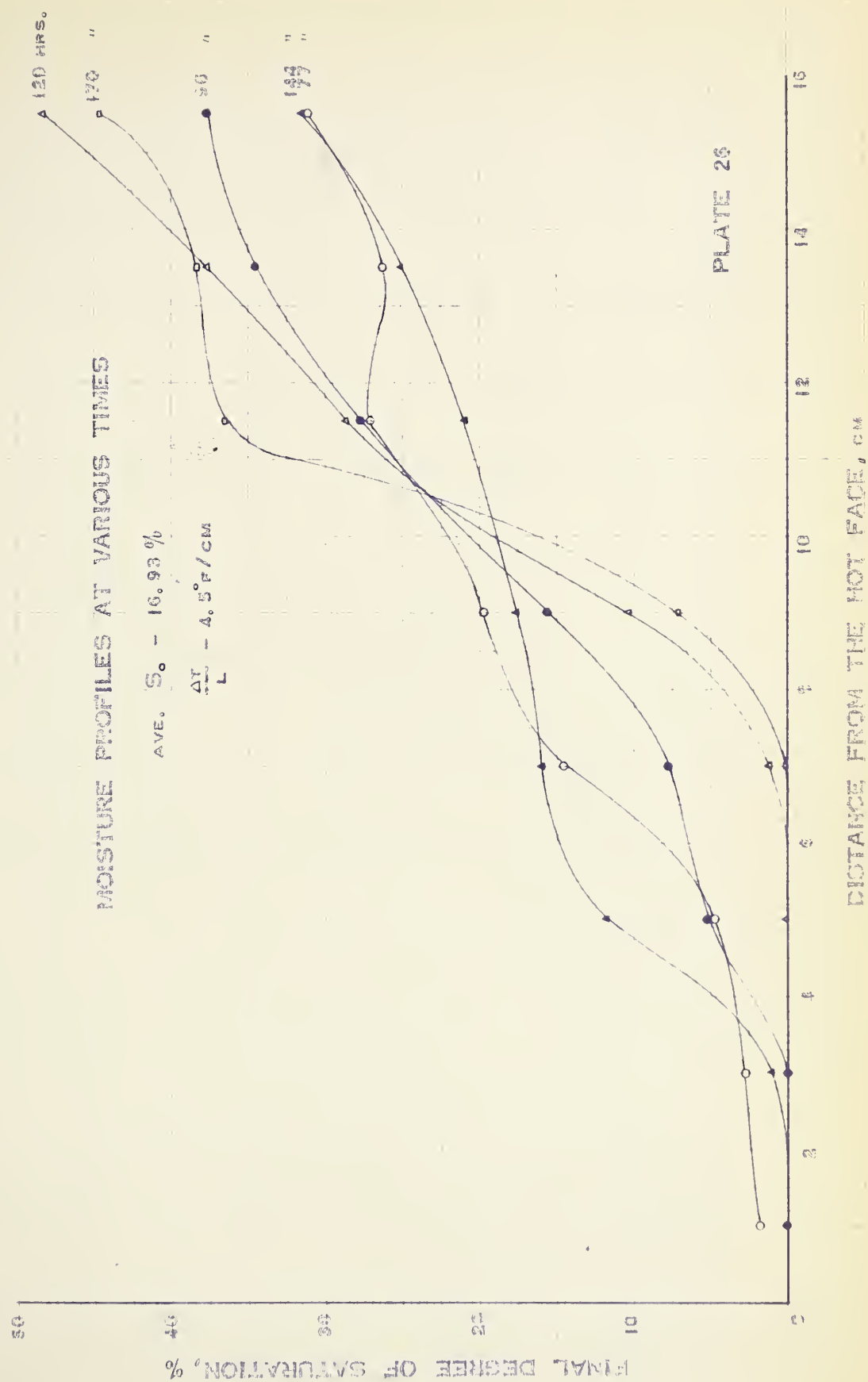
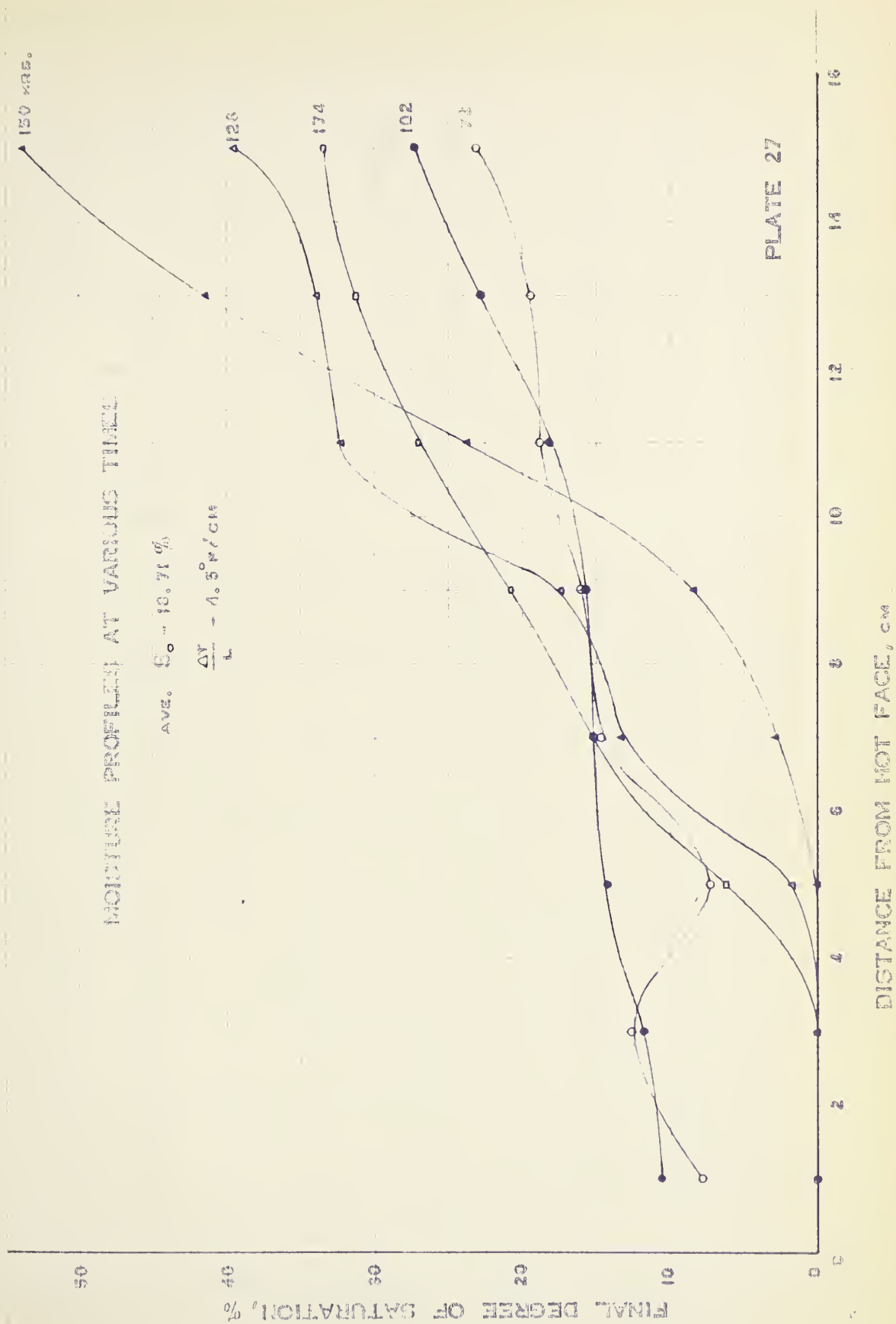


PLATE 25





MOISTURE PROFILES AT VARIOUS TIMES

AVE. $S_o = 23.77\%$

$$\frac{\Delta T}{L} = 4.5^\circ \text{F}/\text{cm}$$

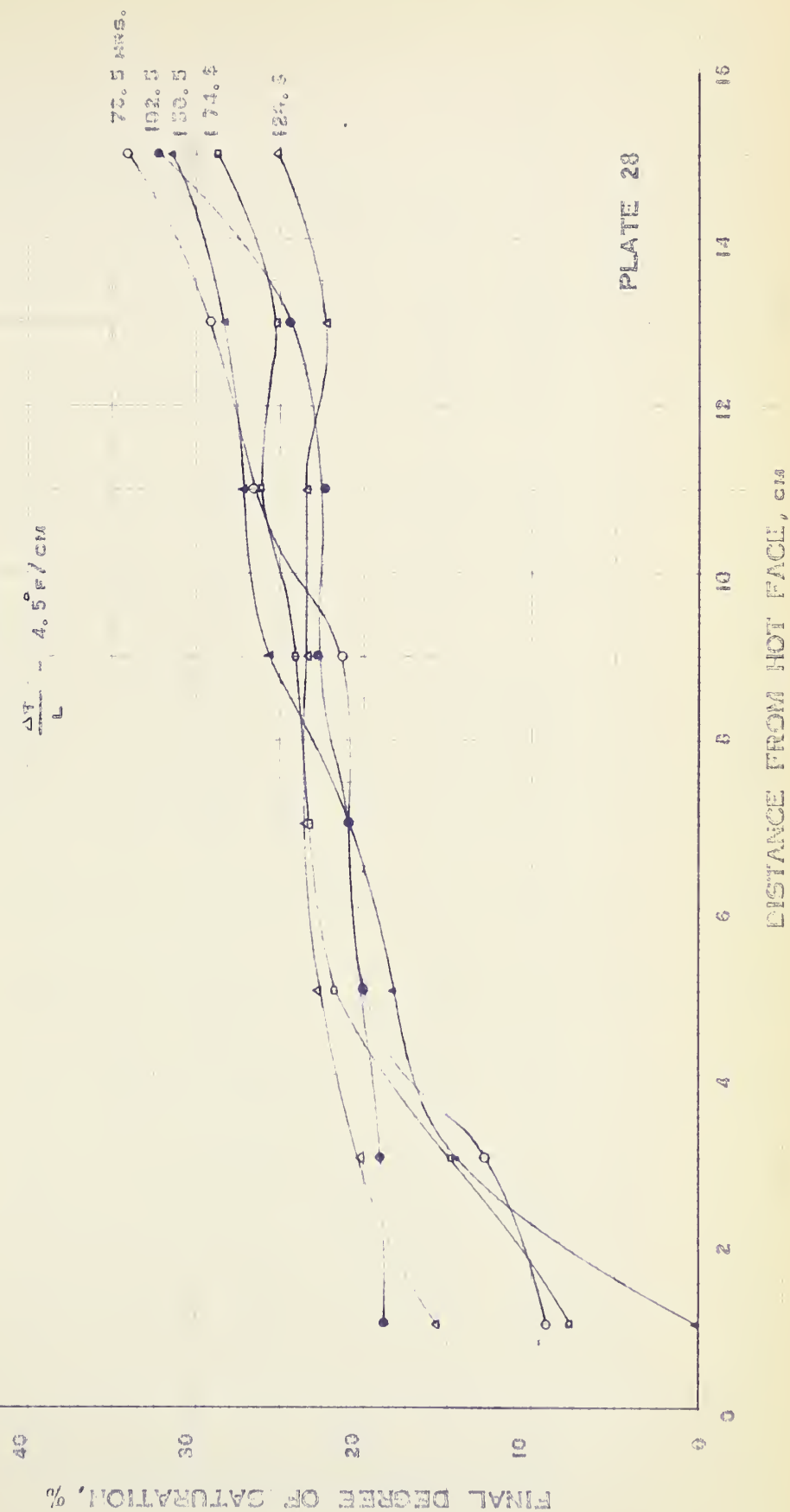


PLATE 28

MOISTURE PROFILES AT VARIOUS TIMES

AVE. $S_0 = 30.91\%$

$$\frac{\Delta T}{L} = 4.5^\circ\text{F}/\text{CM}$$

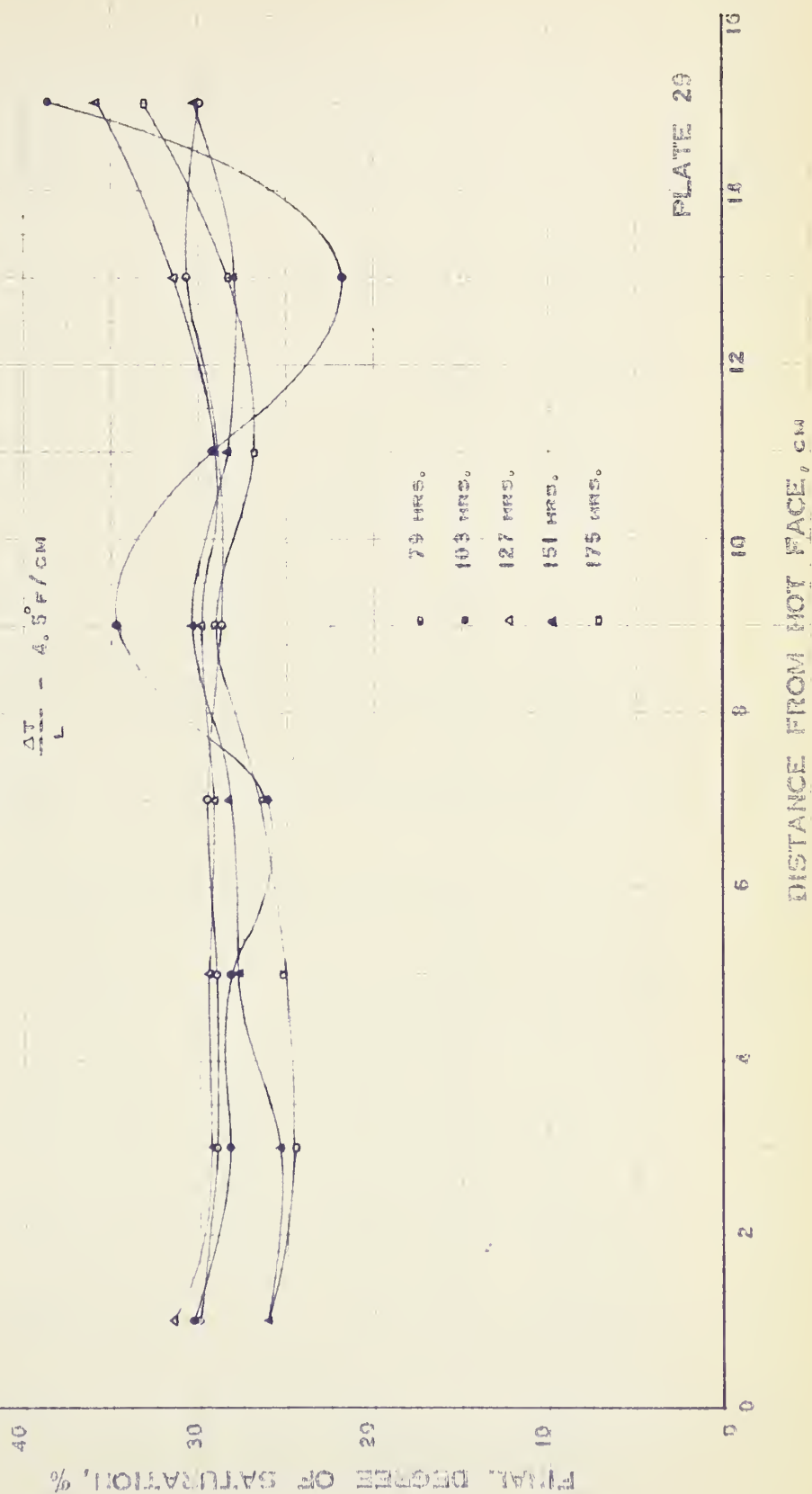


PLATE 29

MOISTURE PROFILES AT VARIOUS TIMES

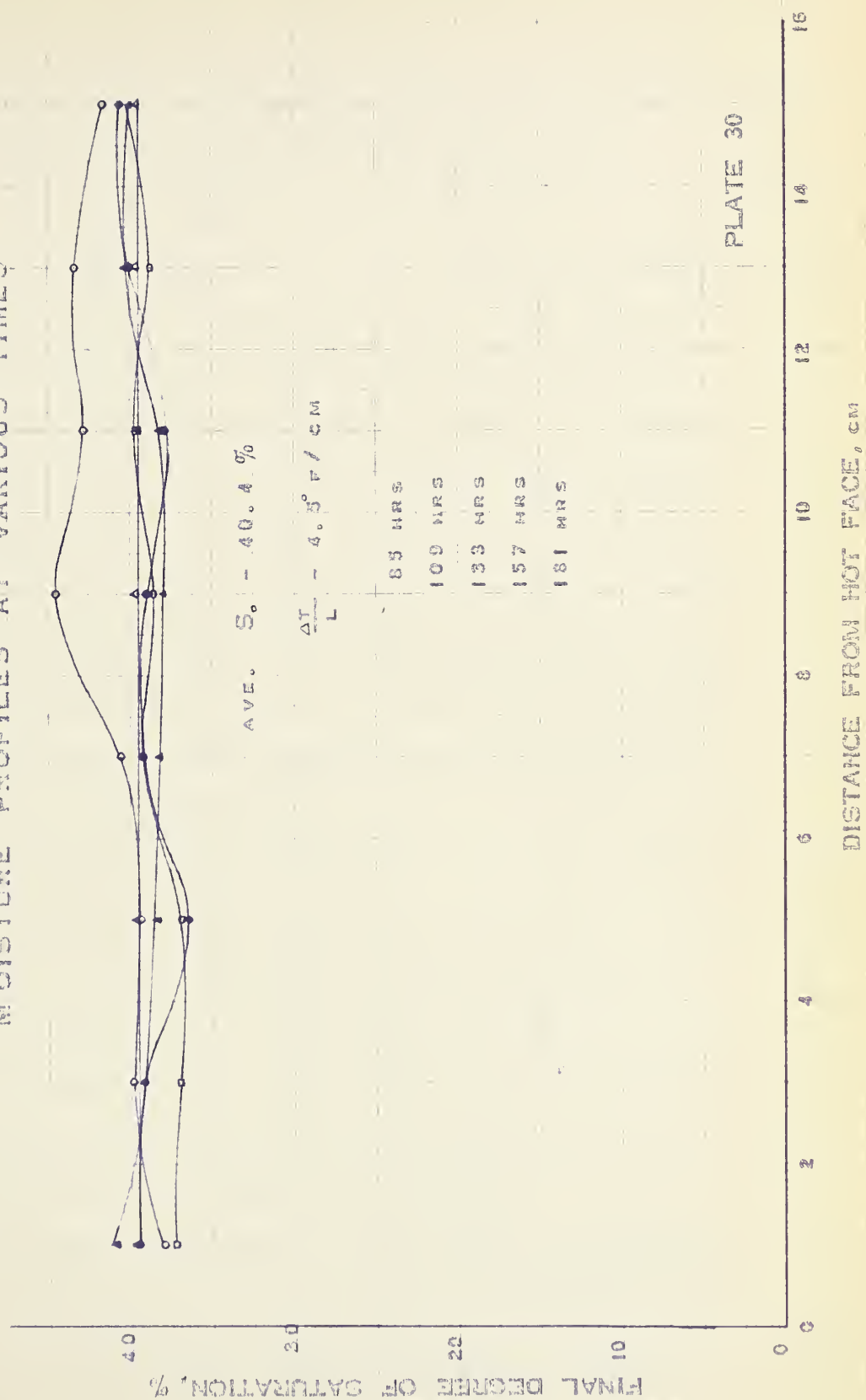


PLATE 30

saturation, but sampled after different elapsed times, have been grouped together. Besides showing that there is an accumulation of water at the cold end in the case of "dry" specimens, these graphs also reveal that a recirculation from the cold to the hot side must be taking place.

6.4.1. For the higher temperature gradient:

The general features of these families of curves (Plates 25-30) are as would be expected in the light of the previous discussions. In the "dry" specimens, the zone of drying progressively extends towards the cold end, while the moisture profiles of the colder sections show marked fluctuations; this is the case in the first two families of curves (initial degrees of saturation 7.8 and 16.9 per cent respectively). In the third family of curves, the extent of the zone of drying also undergoes fluctuations, as is evident by comparing the profiles corresponding to elapsed times of 150 and 174 hours. With further increase in initial degree of saturation, the zone of fluctuation extends progressively towards the hot end, the zone of drying retreats in the same direction and finally disappears. A final point observable is that the range of fluctuation at any section also decreases with increasing degrees of initial saturation.

6.4.2. For the smaller temperature gradient:

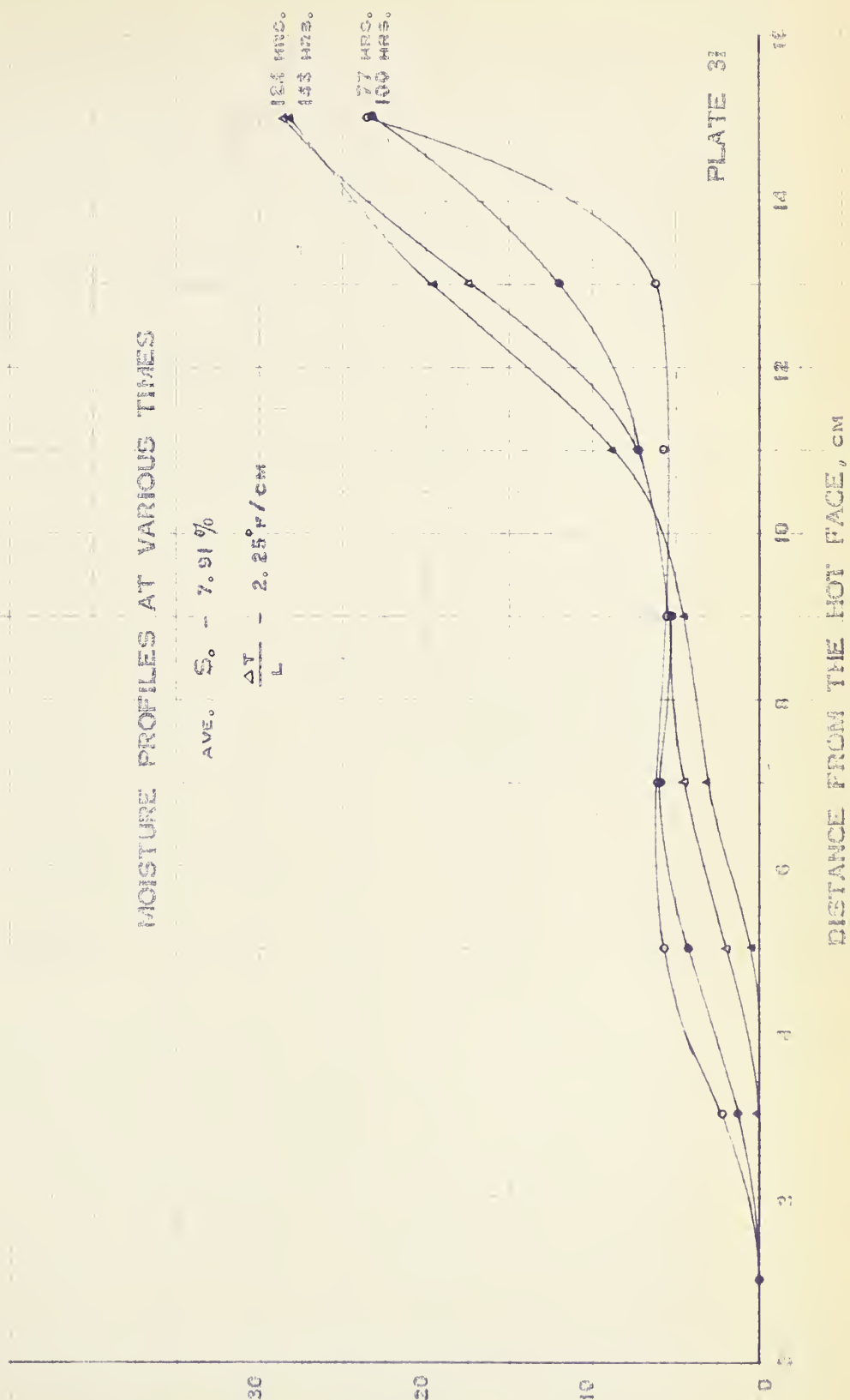
The general features of these curves (31-36) are identical with the ones described in the preceding section.

MOISTURE PROFILES AT VARIOUS TIMES

AVE. $S_o = 7.91\%$

$$\frac{\Delta T}{L} = 2.25^\circ\text{F}/\text{cm}$$

FINAL DEGREE OF SATURATION, %



MOISTURE PROFILES AT VARIOUS TIMES

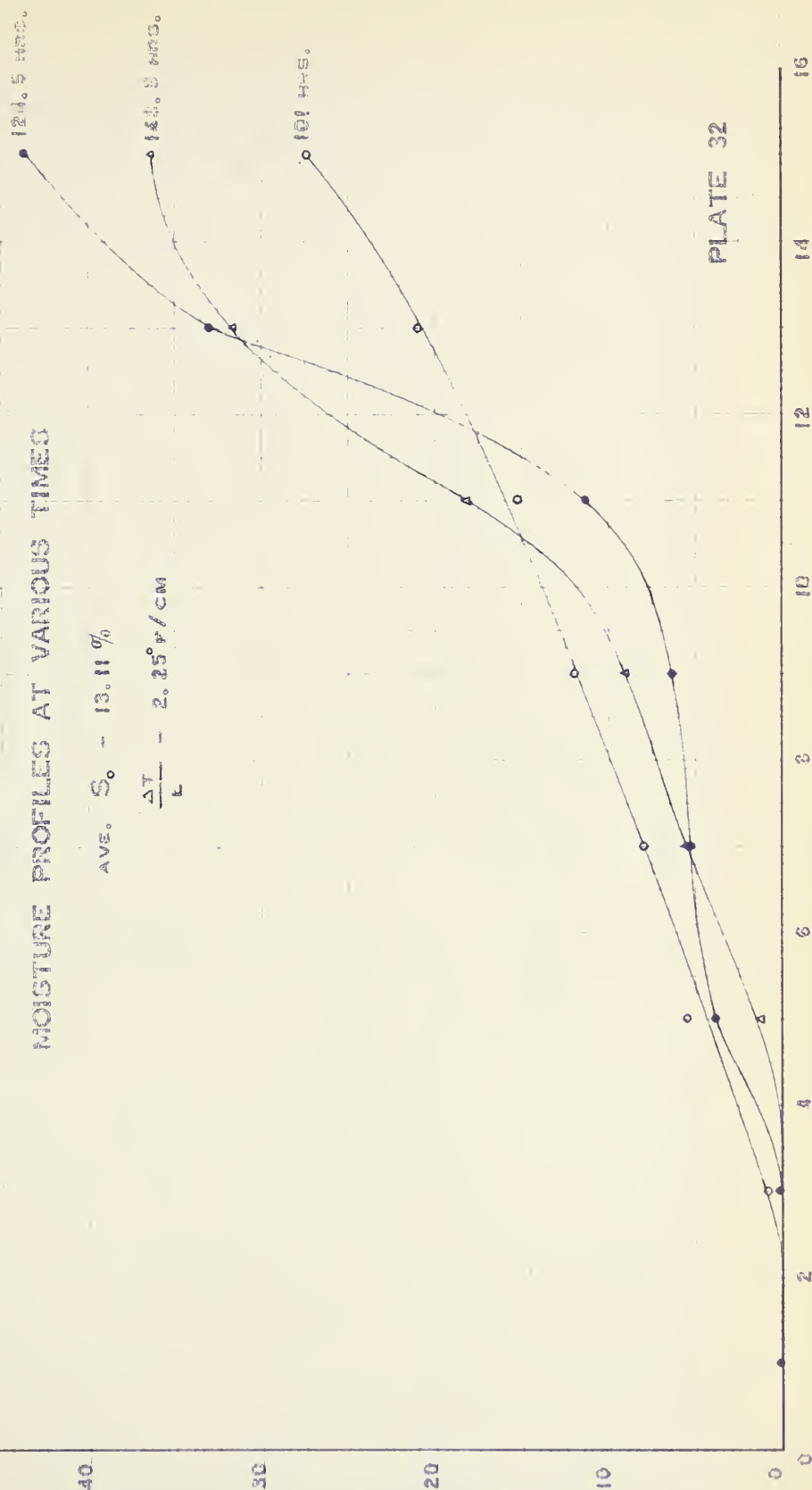
AVE. $S_o = 13.11\%$

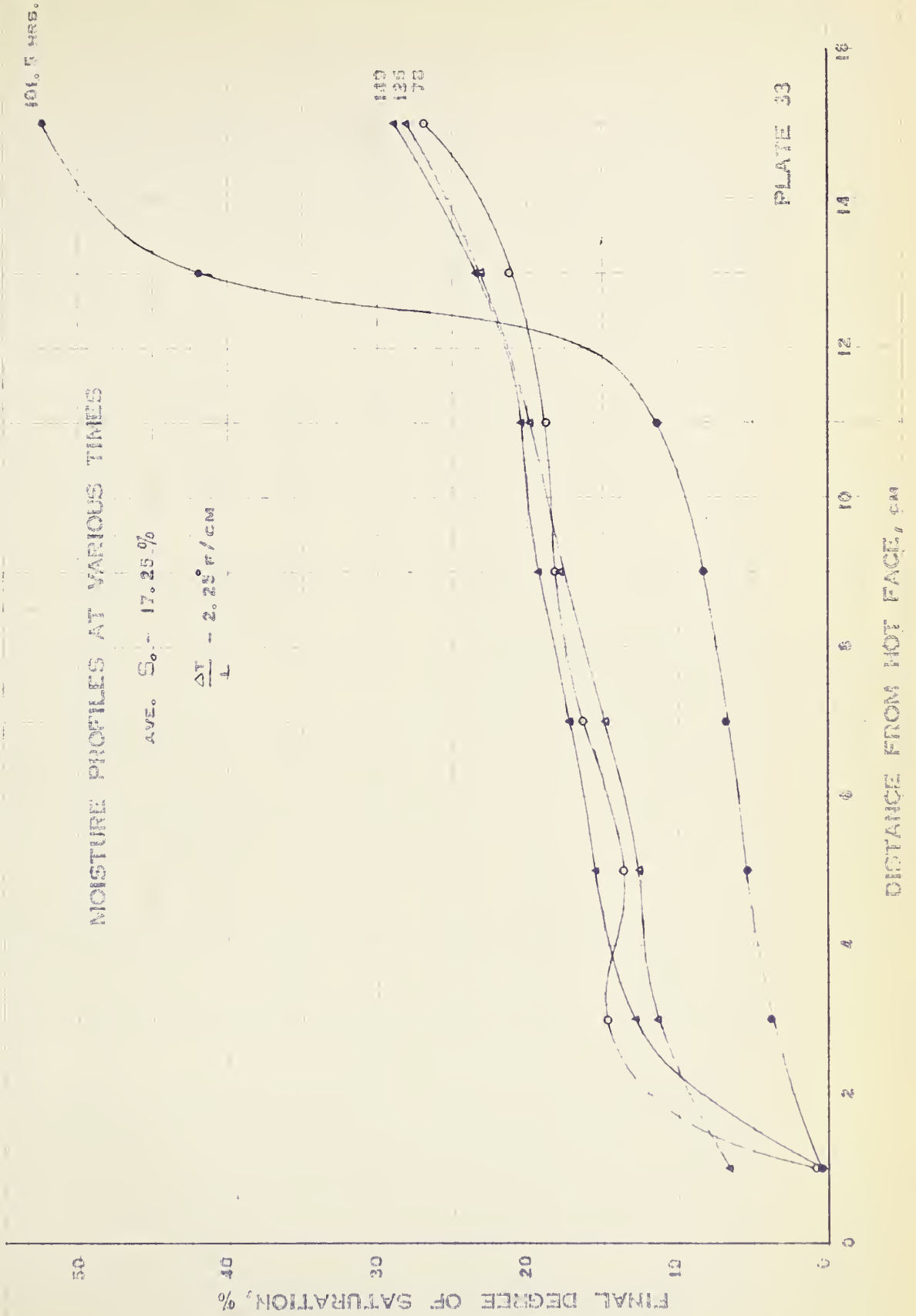
$\frac{\Delta T}{L} = 2.25^\circ\text{F}/\text{CM}$

FINAL DEGREE OF SATURATION, %

PLATE 32

DISTANCE FROM THE HOT FACE, CM





MOISTURE PROFILES AT VARIOUS TIMES

AVE. $S_0 = 25.27\%$

$$\frac{\Delta T}{L} = 2.25^\circ\text{F}/\text{cm}$$

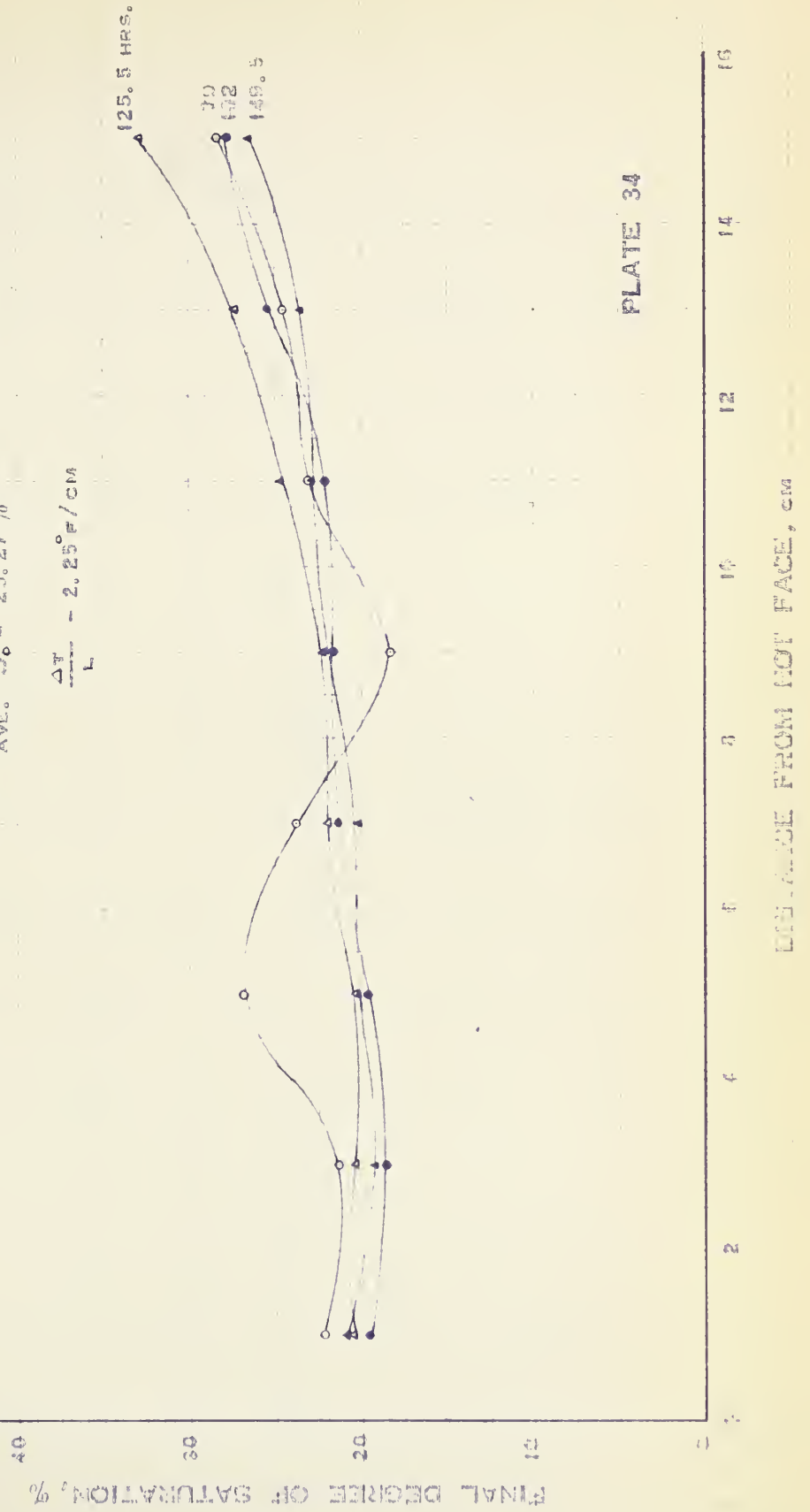
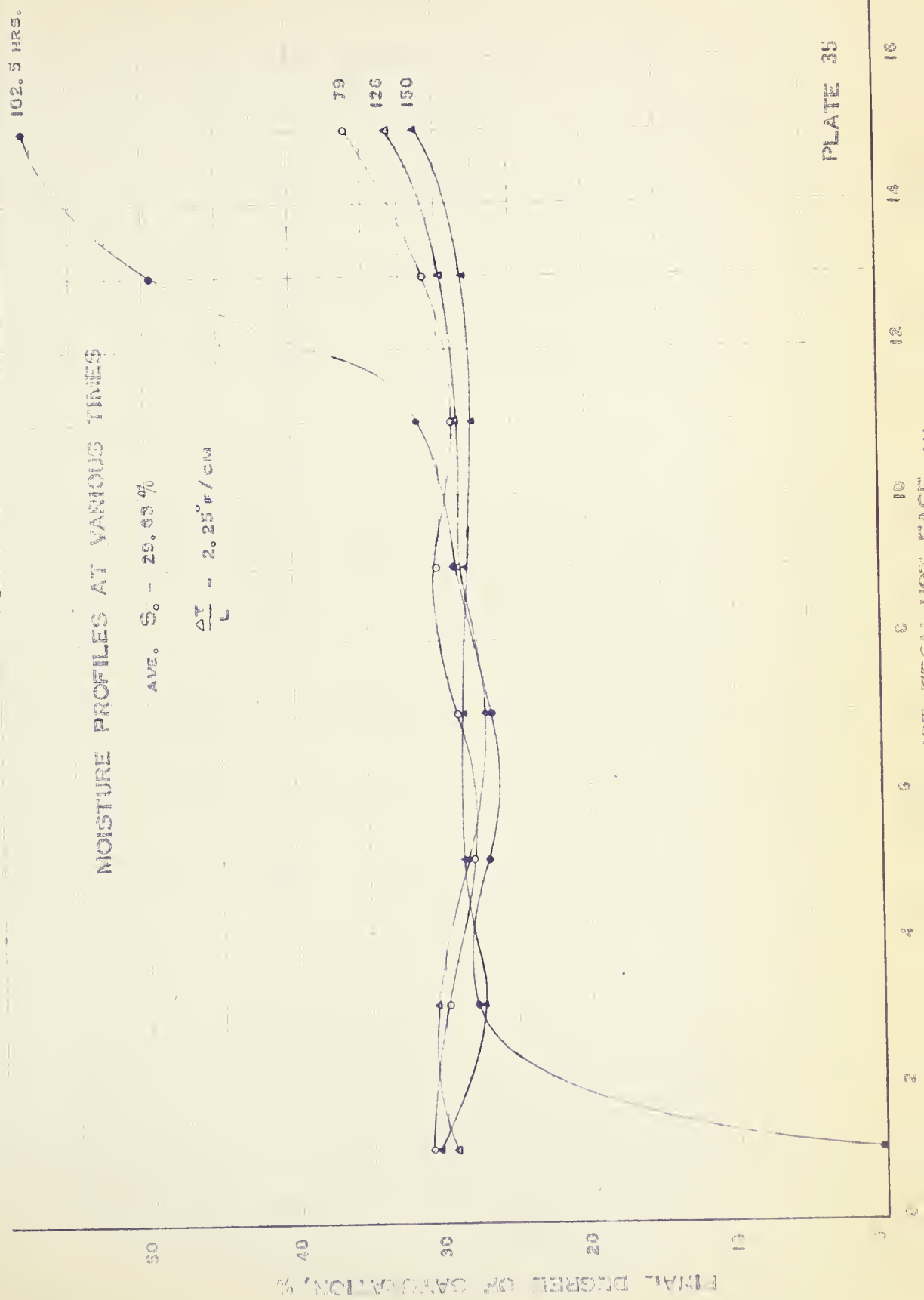


PLATE 34



MOISTURE PROFILES AT VARIOUS TIMES

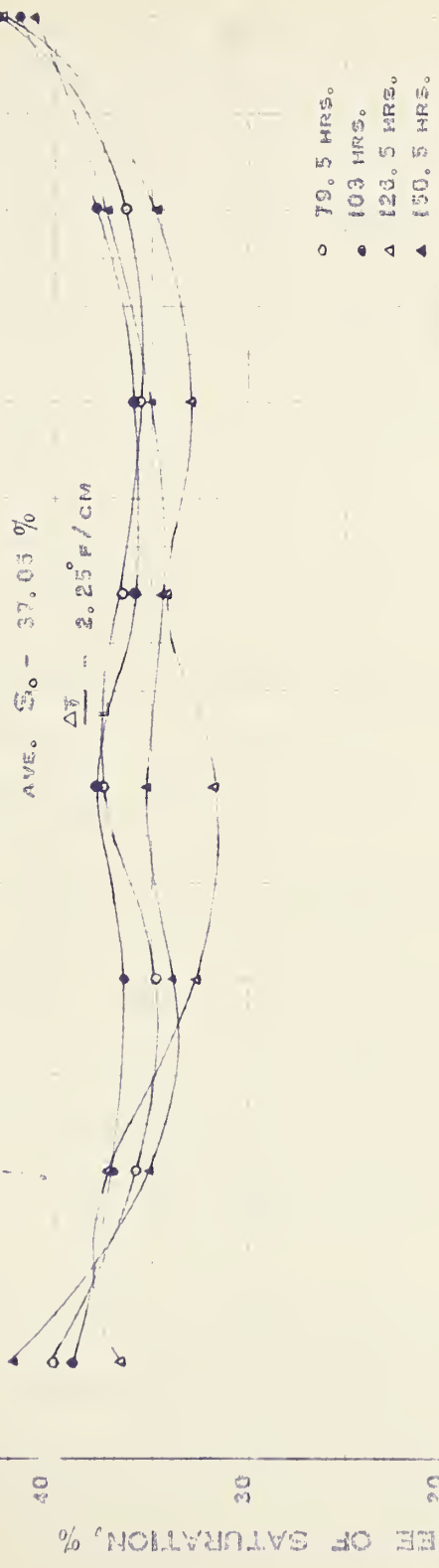


PLATE 36

DISTANCE FROM HOT FACE, CM

The only anomaly is observed in the family of curves corresponding to the initial degree of saturation of 29.3 per cent (Plate 35). As is seen from these curves, the profiles after elapsed times of 79, 126 and 150 hours are not materially different from one another, while the profile after 102.5 hours shows a marked drying at the hot end and a considerably greater accumulation of water at the cold end; the intermediate sections have profiles similar to those of the rest of the specimens of this group. As mentioned in an earlier section (cf. section 6.2.2), this situation is inexplicable.

That such a state of instability does exist is brought out by comparing the graph of the similar group of specimens (initial degree of saturation 30.9 per cent), subjected to a temperature gradient of 4.5°F./cm. (see Plate 29). In this case also, the profile at an elapsed time of 103 hours shows larger fluctuations than the profiles at other times. As both groups of specimens were not tested over a sufficiently long period of time, we are not in a position to state definitely that such an instability will occur after an elapsed time of 204 hours also; but such a possibility cannot be excluded altogether.

If it is established that such a large accumulation does take place periodically, it will be impossible to account for it from the concept of vapour flow alone,

because, in this range of values of saturation, the air-filled porosity and the amount of continuous air voids are both small and it is inconceivable that such a large quantity of water could have been transported in the vapour phase.

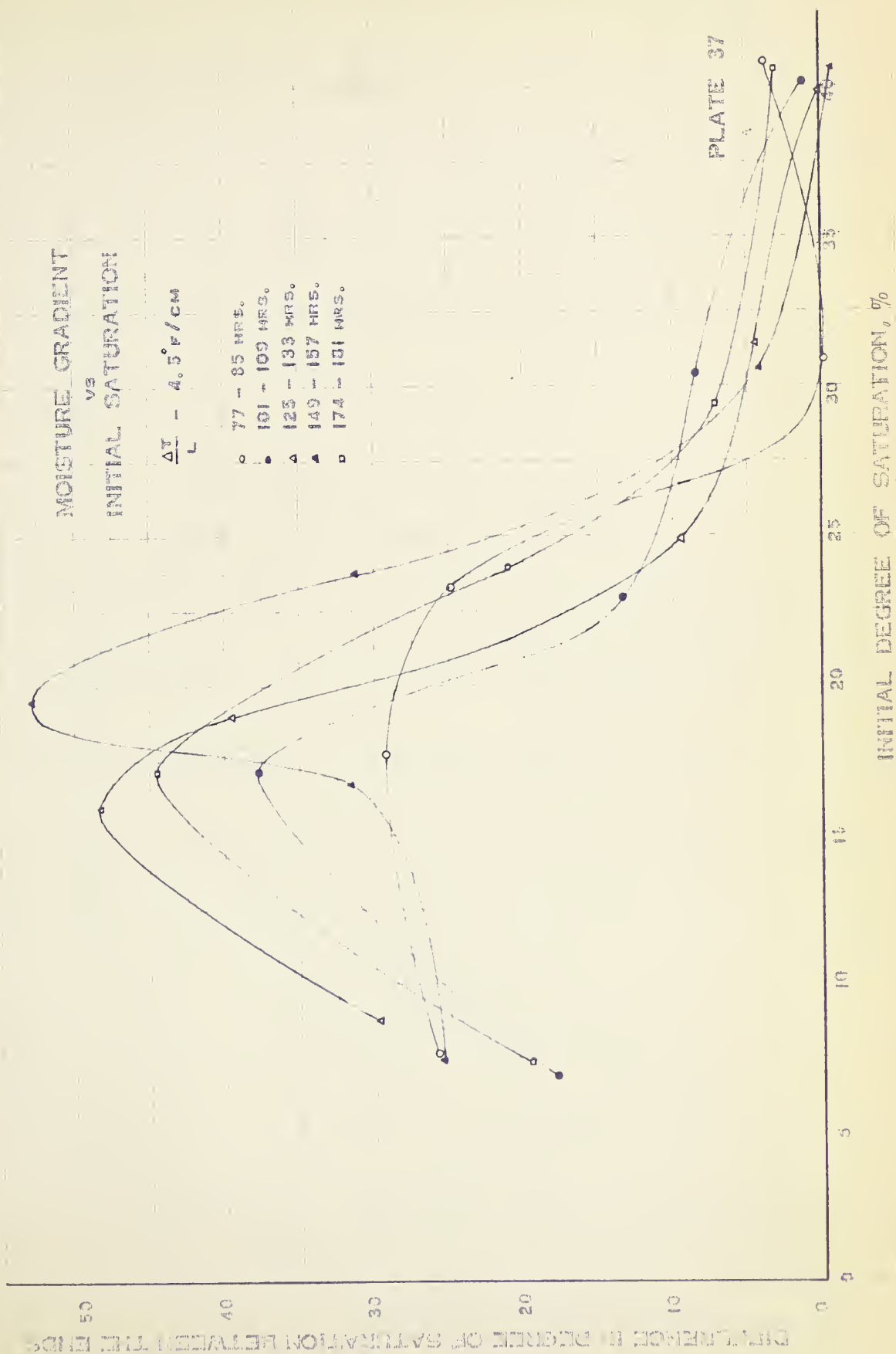
6.5. Moisture gradients vs. initial saturation:

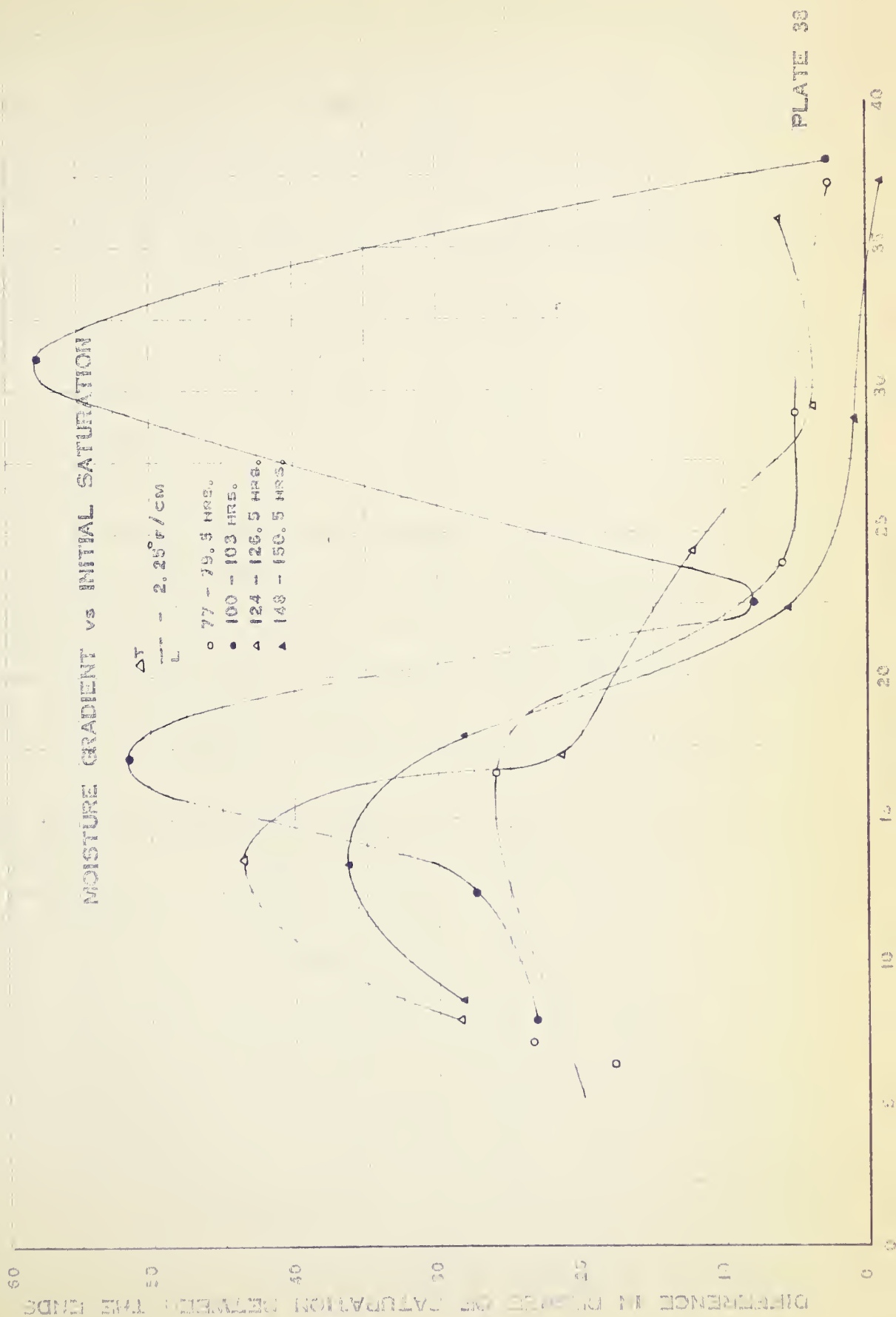
The variation in the moisture concentration gradient between the cold and hot ends of the specimen with the initial degree of saturation, at various elapsed times is illustrated by Plates 37 and 38. The existence of an optimum initial degree of saturation for maximum gradient is quite apparent from these graphs.

Of necessity, the moisture concentration gradient has been assumed to be uniform across the length of the specimen and, accordingly, the ordinates of these graphs represent the difference in the final degree of saturation between the cold and the hot ends of the specimen. It is quite obvious, as the preceding sets of graphs have shown, that the gradient may be much steeper over some portions of the specimen, than the overall gradient. However, as the portions having such steep gradients are not the same for different specimens, it is not possible to compare the individual gradients with gradients over homologous sections of other specimens.

6.5.1. For the higher temperature gradient:

The first of these graphs (Plate 37) represents the data obtained with a temperature gradient of 4.5°F./cm.





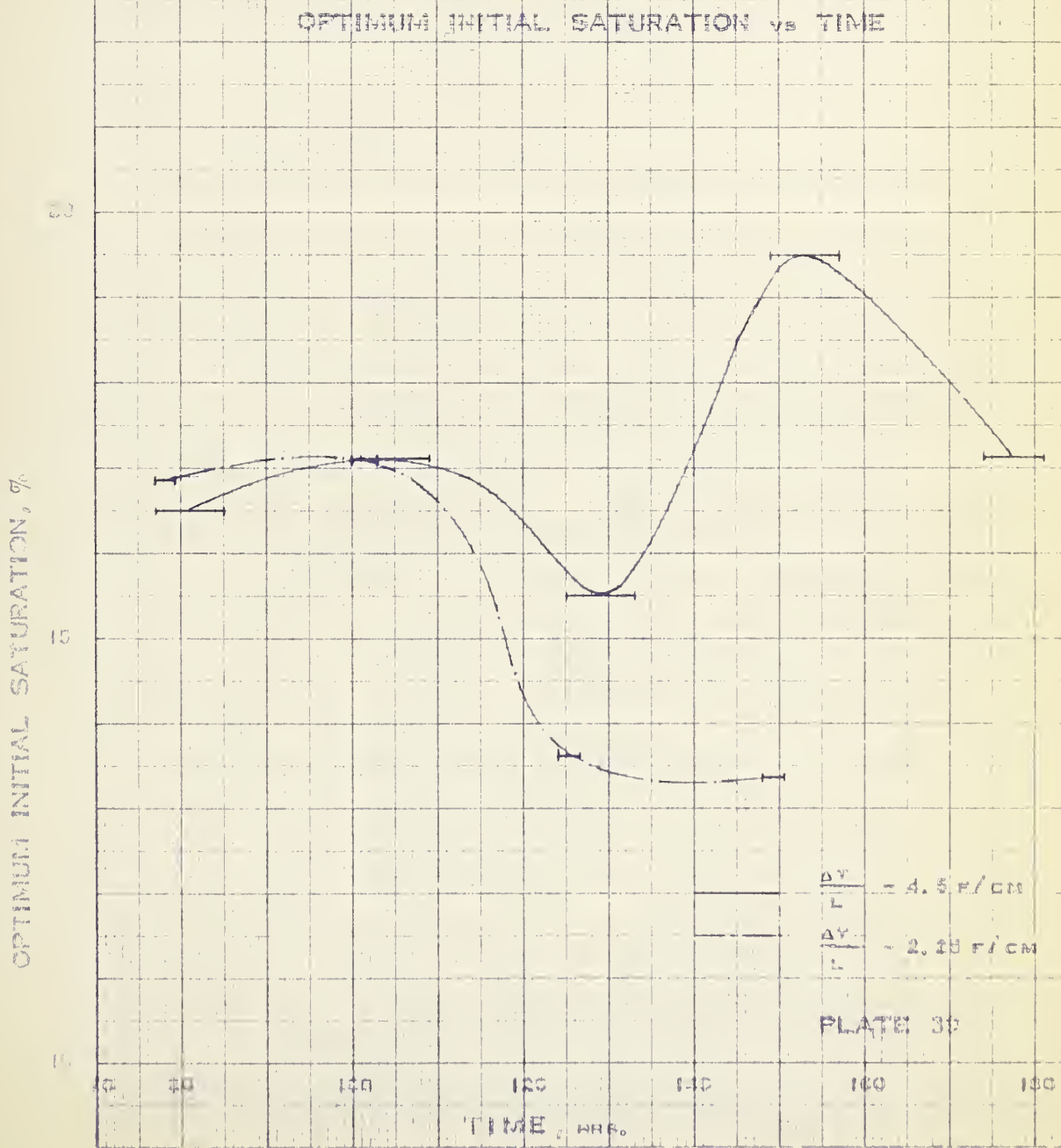
The 'optimum' initial degree of saturation is found to vary between 15 and 20 per cent. The influence of the initial degree of saturation in governing the net transfer of water from the hot to the cold side is once again apparent; in particular, the difference in degree of saturation between the ends is never greater than 10 per cent, for all specimens with an initial degree of saturation greater than about 28 per cent.

6.5.2. For the smaller temperature gradient:

In this case (Plate 38), the optimum initial degree of saturation is seen to range between 13.5 and 17.2 per cent. The anomalous existence of two optimum degrees of saturation for the specimens subjected to a temperature gradient for 100-103 hours has already been discussed. The lower optimum value has been used in the computation of the range of variation mentioned above.

6.5.3. Optimum degree of saturation vs. time:

In view of the fact that, in both the series of tests, the optimum initial degree of saturation varies with time, it is of interest to ascertain whether the variation shows any regular trend or not. For this purpose, the optimum initial degrees of saturation, obtained from the above discussed curves, have been plotted against time (Plate 39). As will be seen from these two curves (one each for each temperature gradient),

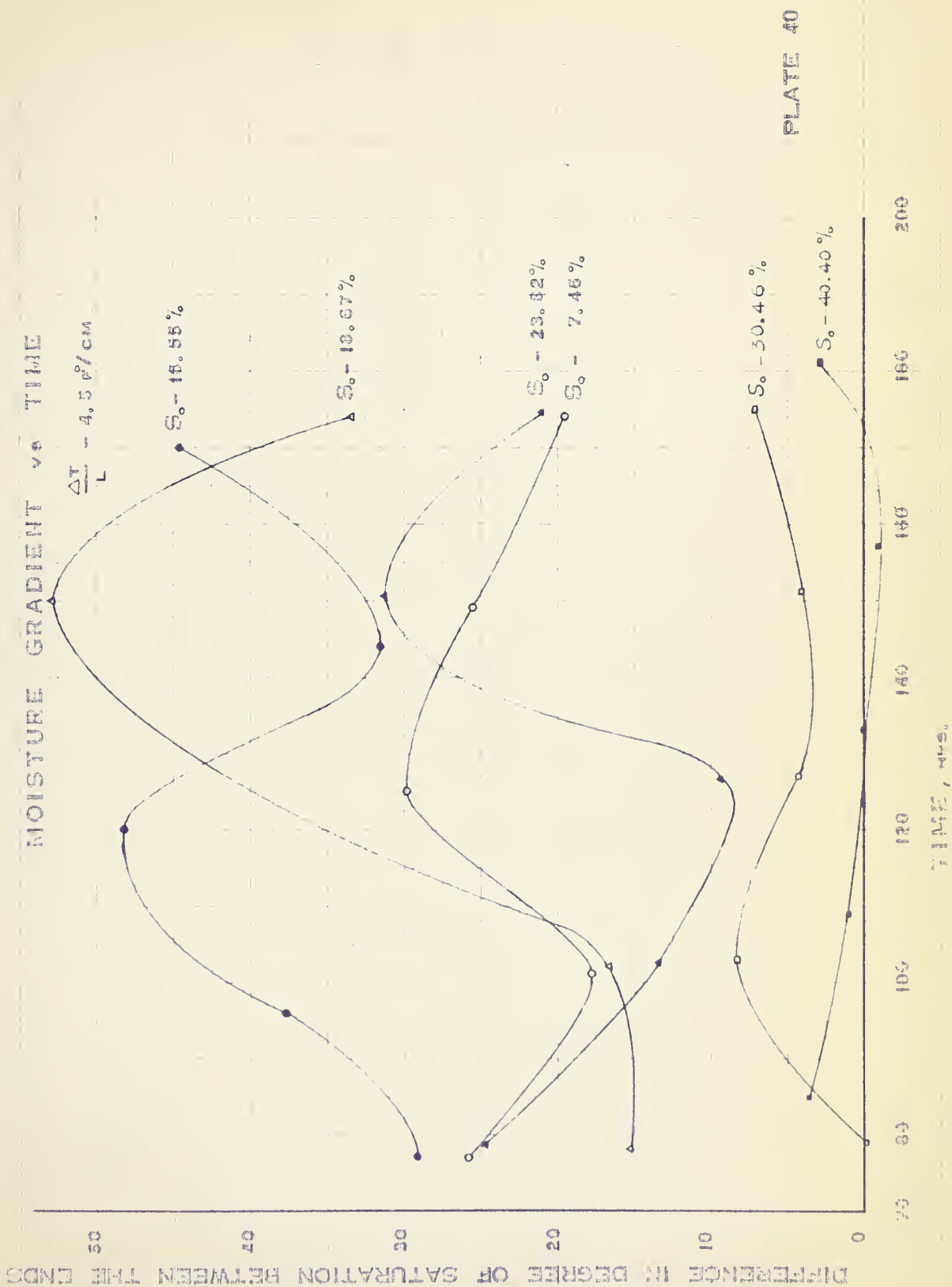


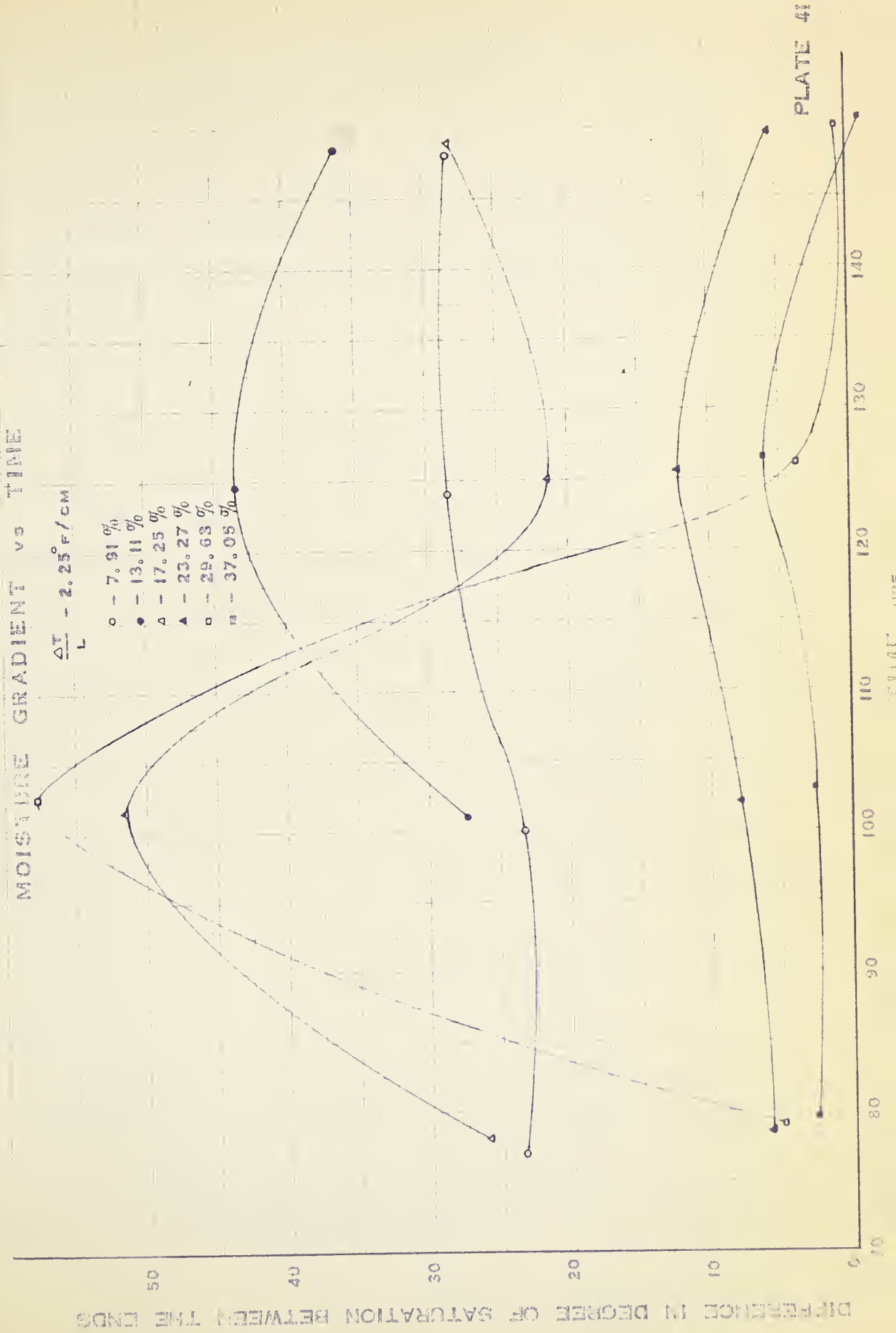
the variation in the optimum value with time does follow a regular pattern.

While limited data forbid us to reach any definite conclusions of general validity, it seems reasonable to hypothesize that this variation in the optimum initial degree of saturation is also due to the transient nature of the flow; when all the specimens have reached a state of dynamic equilibrium, this variation will likely disappear.

6.6. Moisture gradient vs. time:

The same data have been plotted in a different manner, in order to bring out the time-dependent nature of the phenomenon: the elapsed time has been chosen as the independent variable; the data obtained from specimens having nearly the same initial degree of saturation have been grouped together. Both sets of graphs of this series (Plates 40 and 41) show clearly that the moisture concentration gradient varies with time and that it attains maxima and minima; of particular interest is the fact that the abscissae of the points of inflexion of the various members of the family of curves are not equal. Thus, it is seen that the conditions in the various specimens are different from one another at any one time; while the conditions in one specimen are such as to give a maximum moisture gradient from the cold to the hot side, the conditions in another specimen, for the same elapsed time, are tending towards a minimum value of the gradient and so on.





The final point worthy of mention is that the gradient assumes small negative values in the wetter specimens after certain elapsed times, thus indicating an accumulation of water at the hot end also. As will be seen from the graphs, this accumulation is of a temporary nature only. Even so, such an accumulation at the hot end could not have taken place, unless there was a return flow of water towards the hot end.

6.7. Concluding remarks:

The data obtained from these tests show very clearly that water does flow back from the cold to the hot side, the extent of the flow region depending upon the distance up to which continuous water films are present. As regards the primary mechanism involved in the thermo-osmotic flow, it appears probable that it is comprised of a vapour flow across the empty pores followed by liquid flow through the contact menisci. The following considerations lend support to this tentative conclusion:

(1) It has been shown that the temperature does not appreciably affect the P_F of the soil water at unaltered water contents (Croney and Coleman, 1948); so, the flow cannot be wholly accounted for by the thermally-induced P_F gradient.

(2) Except in very dry soils, the soil moisture will be distributed as held water around contact wedges and so flow cannot be exclusively in the vapour phase since continuous air voids are not present.

As regards the existence of an optimum initial degree of saturation for maximum thermo-osmotic flow, the data do reveal that such is indeed the case, in accordance with the findings of the earlier workers in the field. But, there seems to be no general measure of agreement between the reported order of magnitudes of this optimum value: Bouyoucos reportedly states that the value is close to the plastic limit of the soil, whereas the data of Habib (1957) suggest that the optimum value is far below the plastic limit of the soil. Winterkorn (1947, 1955a) states that the value lies in the vicinity of the Proctor optimum moisture content for maximum densification, while Hutcheon (1958) reports that the value lies between 50 and 60 per cent of the 3/10 atmosphere percentage for the soil.*

For the soil used in the present series of tests, the optimum initial degree of saturation is found to range from 15 to 20 per cent for the higher temperature gradient of 4.5°F./cm., and from 13.5 to 17.2 per cent for the smaller temperature gradient of 2.25°F./cm.; based on an average void ratio of 0.72 for all the specimens tested, it is seen that the air-filled porosities range from 32 to 38 per cent of the volume of the soil. Habib reports that the air-filled porosity in his experiments was 26%. There are two probable reasons for this discrepancy between the

* The 3/10 atmosphere percentage is approximately equal to the field moisture equivalent of the soil.

two sets of data: in the first place, the soils used in the two cases are vastly different: Habib used a plateau loam of low plasticity ($w_1 = 34\%$, $w_p = 18\%$, $I_p = 16\%$), while the present experiments were conducted using a nonplastic, closely-graded sample of fine silica sand. Secondly, since the present series of experiments were primarily concerned with the transient stages of thermo-osmotic flow, the specimens had not attained a state of dynamic equilibrium, while it is possible that Habib's results are based on data obtained from specimens after a stable distribution of moisture was established. It is believed that the results of the present series of tests would have been lowered somewhat, if the specimens had been allowed to reach a state of equilibrium.

For the soil tested, the optimum moisture content for maximum density determined by the Standard Proctor test is 4.85 per cent, corresponding to a degree of saturation of 17.3 per cent. It will be noticed that this value is within the range of variation of the optimum initial degree of saturation for maximum thermo-osmotic transfer for the same soil. Thus, the experimental data are in accordance with Winterkorn's contention that the optimum moisture content for maximum thermo-osmotic flow falls close to the Proctor optimum moisture content (Winterkorn 1947).

C H A P T E R VII

CONCLUSIONS AND RECOMMENDATIONS

7.1. General conclusions:

1. The P_F concept seems to be very valuable in dealing with problems involving accumulation of water beneath impervious pavements; successful applications of the concept have so far been restricted to shallow water table conditions and to cases involving liquid flow only.

2. Under favourable conditions of soil moisture, considerable quantities of water may migrate under the influence of a temperature gradient; the flow is from the hot to the cold side.

3. For a given soil, there exists an optimum moisture content at which thermally-induced transfer of water from the hot to the cold side is a maximum.

4. The overall phenomenon of thermo-osmosis through soils is still imperfectly understood; many controversial hypotheses have been put forward by the various investigators to explain the observed phenomena.

7.2. Conclusions from the present series of experiments:

1. The experimental data indicate that water flowing from the hot to the cold side under a temperature gradient tends to redistribute itself in the liquid phase; the distance up to which such a recirculation takes place depends upon the presence of continuous conducting water films. The zone of fluctuation (where the thermo-osmotic flow and the opposing liquid flow take place at the same time) increases with increasing initial degree of saturation of the specimen.

2. The existence of an optimum initial moisture content (or initial degree of saturation) for maximum net transfer of water from the cold to the hot side has been verified. This optimum degree of saturation appears to vary slightly with the magnitude of the temperature gradient. A unique optimum value for any given temperature gradient can be obtained only when the individual specimens have reached a state of dynamic equilibrium.

3. For the soil tested, the optimum initial degree of saturation is found to vary between 15 and 19.5 per cent for a temperature gradient of 4.5°F./cm. ($38\text{--}110^{\circ}\text{F.}$), and between 13.5 and 17.2 per cent for a temperature gradient of $2.25^{\circ}\text{F./cm.}$ ($42\text{--}78^{\circ}\text{F.}$); this variation may be ascribed to the fact that the specimens had not reached a state of dynamic equilibrium.

4. There exists a particular value of initial degree of saturation beyond which the net transfer of water is negligible; for the soil tested, this value is about 28 per cent.

5. In specimens having an initial degree of saturation greater than 30 per cent, the experiments reveal that a temporary accumulation of water may take place at the hot end also; this condition appears to be a direct consequence of the existence of a return flow of water from the cold to the hot end.

6. The data indicate that the mechanism of thermo-osmosis involves vapour flow across air-filled pores and liquid flow through the contact wedges of held water. Upon the establishment of a suction gradient of sufficient magnitude, the accumulated water at the cold end tends to redistribute itself by flowing towards the hot end, as far as continuous water films are present. In very moist specimens, a state of dynamic equilibrium may never be reached. Thus, the experimental data are in accordance with the findings of earlier workers like Gurr et al., Taylor and Cavazza, and Hutcheon.

7.3. Recommendations for further investigations:

1. Further experimental work in thermo-osmosis may be so designed as to measure the P_F of the soil water at various sections of the specimens as a function of time

and of the initial degree of saturation. Such experiments may prove to be invaluable in many ways: they may throw more light on the actual mechanism involved in thermo-osmotic flow; and, they may also pave the way for the eventual application of the P_F concept to the case of moisture migration in the vapour phase.

Experience shows that, with fairly high temperature gradients, the soil close to the warm end becomes very dry, especially in specimens having low initial moisture contents. The P_F of soil water in such sections is likely to be very high, depending upon the soil type. No suitable techniques for in-place measurement of P_F beyond 3 are available. So, new techniques may have to be developed, or high temperature gradients must be avoided.

2. It is also desirable to conduct these experiments using different types of soils, and ascertain whether there is any correlation between the factors involved in thermo-osmosis on the one hand, and the properties of the soils on the other hand. The former may include the following: maximum thermo-osmotic transfer, thermo-osmotic permeability, optimum initial degree of saturation for maximum flow, and the minimum initial degree of saturation for negligible flow; the latter may include: limits of consistency, optimum moisture contents for maximum densification according to the different specified compaction procedures, base exchange capacity, hygroscopic moisture content and type of soil mineral constituent.

3. The present series of experiments were conducted with closed systems. In the field, the subgrade is usually in communication with a water table below; if the surfacing is freely permeable, the field conditions will approximate to an open system. The case of a subgrade overlain by an impervious pavement belongs to an intermediate state of conditions, if we assume that lateral movement of water from the center of the pavement towards the shoulders is negligible. Thus, it will be of great practical interest to investigate the extent of accumulation at the cold end in a laboratory model simulating these field conditions. A simple testing arrangement for this purpose would seem to be to replace the brass end plug at the hot end by a plug made of porous bronze or similar material; piezometer tips located close to the hot end may serve to indicate the pressure of water at entry, which may be utilized in making allowance for the flow under hydrostatic pressure alone.

4. In view of the fact that it is extremely difficult, if not altogether impossible, to prepare two identical specimens of the same soil, it is desirable that a suitable technique for in-place measurements of moisture contents of laboratory soil samples be evolved. Such a technique will eliminate the need for a large number of specimens having the same initial moisture content and the apparatus

can be simplified considerably. More important, however, is the fact that it will be possible to dispense with the need of removing the specimen from the apparatus and of determining the moisture content by the conventional method of oven drying. By removing the specimen from the apparatus, we alter the temperature conditions across the specimen thus upsetting the vapour equilibrium conditions at the different sections of the specimen; thus liquid water may pass into the vapour phase at the cold end, while the water vapour may condense into the liquid phase at the warm end. Since the relative humidity and the temperature conditions are different for different sections of the same specimen, this difficulty cannot be overcome by employing a constant temperature room or room with controlled humidity conditions for the location of the apparatus. The need for an acceptable method of in-place measurement of the moisture content cannot possibly be overemphasized.

BIBLIOGRAPHY

1. Adam, N.K., "The Physics and Chemistry of Surfaces",
London: University Press, 1941 (III ed.).
2. Bakhmeteff, B.A., and Feodoroff, N.V., "Flow Through
Granular Media", Trans. A.S.M.E., 59: A98-104, 1937.
3. Barrer, R.M., "Fluid Flow in Porous Media", Disc.
Faraday Soc. 3, 1948.
4. Baver, L.D., "Soil Physics", New York: John Wiley and
Son Inc., 1956.
5. Bernatzik, W., "Contribution to the Problem of Seepage
Pressures", Proc. II Int. Conf. Soil Mech. Found.
Engg. 3: 65-66, 1948.
6. Black, W.P.M., and Croney, D., "Pore Water Pressures
and Moisture Content Studies Under Experimental
Pavements", Proc. IV Int. Conf. Soil Mech. Found. Engg.
4: 94-103, 1957.
7. Bodman, G.B., and Colman, E.A., "Moisture and Energy
Conditions During Downward Entry of Water into
Soils", Soil Sci. Soc. Amer. Proc. 8: 116-122, 1944.
8. _____, and Day, P.R., "Freezing Point of a Group
of California Soils, and Their Extracted Clays",
Soil Sci. 55: 225-246, 1942.

9. _____, and Edlefson, N.E., "The Soil Moisture System", Soil Sci. 38: 425-444, 1934.
10. Bolt, G.H., and Miller, R.D., "Calculation of the Total and Component Potentials of Water in Soil", Trans. Amer. Geophys. Union, 39: 916-928, 1958.
11. Bouyoucos, G.J., "Classification and Measurement of the Different Forms of Water in the Soil by Means of the Dilatometer Method", Michigan Agric. College, Expt. St. Tech. Bull. 36, 1917.
12. Brunauer, S., "The Adsorption of Gases and Vapours: I - Physical Adsorption", Princeton: Princeton University Press, 1943.
13. Buckingham, E., "Studies on the Movement of Soil Moisture", U.S.D.A. Bur. Soils, Bull. 38, 1907.
14. Bull, H.B., "Electrokinetic Potential XIV: A Critical Comparison of Electrophoresis, Streaming Potential and Electro-osmosis", J. Phys. Chem. vol. 39, 1935.
15. _____, and Gortner, R.A., "Electrokinetic Potential VI: Electrical Phenomena at Interfaces", J. Phys. Chem. 35: 309-30, 1931.
16. _____, and Moyer, L.S., "Electrokinetic Potential XVI: Streaming Potential in Small Capillaries", J. Phys. Chem. vol. 40, 1936.
17. Butler, J.A.V. (editor), "Electrical Phenomena at Interfaces", London: Methuen and Co. Ltd., 1951 (Chapters II and III).

18. Campbell, R.B., "Freezing Point of Water in Puddled and Unpuddled Soils at Different Soil Moisture Tension Values", Soil Sci., 73: 221-28, 1952.
19. Carman, P.C., "Permeability of Saturated Sands, Soils and Clays", J. Agri. Sci., vol. 29, 1939.
20. _____, "Capillary Rise and Capillary Movement of Moisture in Fine Sands", Soil Sci., 52: 1-14, 1941.
21. _____, "Some Physical Aspects of Water Flow in Porous Media", Discussions of the Faraday Society, vol. 3, 1948.
22. Casagrande, A., "Seepage Through Dams", J. New England Water Works Assoc. 51: 131-172, 1937. Reprinted in "Contributions to Soil Mechanics, I", Boston Soc. Civ. Engrs.
23. Casagrande, L., "Electro-osmosis", Proc. II Int. Conf. Soil Mech. Found. Engg. 1: 210-233, 1948.
24. _____, "Electro-osmotic Stabilization of Soils", J. Boston Soc. Civ. Engrs. 39, 1952.
25. _____, "Review of Past and Current Work on Electro-osmotic Stabilization of Soils", Harvard University, Soil Mechanics Series No.
26. Childs, E.C., "The Transport of Water Through Heavy Clay Soils: I", J. Agri. Sci., 26: 114-127, 1936.
27. _____, "The Transport of Water Through Heavy Clay Soils: II", J. Agri. Sci., 26: 527-545, 1936.

28. _____ , "The Movement of Water in Heavy Clay
Soils After Irrigation", Soil Sci., 46: 95-106, 1938.
29. _____ , "The Water Table, Equipotentials and
Streamlines in Drained Land: II", Soil Sci.
59: 313-327, 1945.
30. _____ , and George, N.C., "Soil Geometry and
Soil-Water Equilibria", Disc. Faraday Soc. 3, 1948.
31. _____ , and _____ , "The Permeability of
Porous Materials", Proc. Royal Soc. London,
A201: 392, 1950 (a) .
32. _____ , and _____ , "Movement of Moisture
in Unsaturated Soils", Trans. IV Int. Cong. Soil
Science (Amsterdam), I: 60-64, 1950 (b) .
33. _____ , and _____ , "The Control of Soil
Water", Advances in Agronomy, 2: 233-272, 1950 (c) .
34. Colman, E.A., and Bodman, G.B., "Moisture and Energy
Conditions During Downward Entry of Water into Moist
and Layered Soils", Soil Sci., Soc. Amer. Proc.
9: 3-11, 1945.
35. Coleman, J.D., "Soil Thermodynamics and Road
Engineering", Nature, vol. 163, 1949.
36. _____ , "An Investigation of the Pressure
Membrane Method for Measuring the Suction Properties
of Soils", DSIR (England) R.R.L., RN/3464/JDC, April 1959.
37. Crawford, C.B., and Legget, R.F., "Ground Temperature
Investigations in Canada", Eng. J. 40 (3), March, 1957.

38. Croney, D., "Movement and Distribution of Water in Soils", *Geotechnique*, vol. 3, 1952.
39. _____, and Coleman, J.D., "Soil Thermodynamics Applied to Movement of Moisture in Road Foundations", *Proc. VIII Int. Cong. Applied Mech.* 3: 163-177, 1948.
40. _____, and _____, "Soil Structure in Relation to Soil Suction (p_F)", *J. Soil Sci.*, vol. 5, No. 1: 75-84, 1954.
41. _____, _____, and Black, W.P.M., "Movement and Distribution of Water in Soil in Relation to Highway Design and Performance", *Highway Res. Board*, SR 40: 226-252, 1958.
42. _____, _____, and Bridge, P.M., "The Measurement of Suction of Moisture Held in Porous Materials and Its Relationship to Moisture Content", *D.S.I.R. (England), R.R.L., RN/1528/DC.JDC.PMB.*, March, 1951.
43. _____, _____, and Currer, E.W.H., "The Electrical Resistance Method of Measuring Soil Moisture", *British J. Applied Phys.* 2: 85-91, 1951.
44. Derjaquin, B.V., and Melnikova, N.K., "Mechanism of Moisture Equilibrium and Migration in Soils", *Highway Res. Board*, SR 40: 43-54, 1958.
45. deVries, D.A., "Some Remarks About Heat Transfer by Vapour Movement in Soils", *Trans. IV Int. Cong. Soil Science*, vol. 2, 1950.

46. _____, and Philip, J.R., "Temperature Distribution and Moisture Transfer in Porous Materials", J. Geophys. Research, vol. 64, 3: 386-388, March, 1959.
47. Dhawan, C.L., Verma, B., and Handa, B.K., "Deduction of Schofield Formula from Kelvin Equation", J. Ind. Soc. Soil Sci., 4: 61-63, 1956.
48. Dawson, R.F., and McDonald, R.W., "Some Effects of Electric Current on the Consolidation Characteristics of Soils", Proc. II Int. Conf. Soil Mech. Found. Engg. 5: 51-57, 1948.
49. Edlefson, N.E., "A New Method of Measuring Aqueous Vapour Pressure of Soils", Soil Sci., 38: 29-37, 1934.
50. _____, and Anderson, A.B.C., "Thermodynamics of Soil Moisture", Hilgardia, 15: 31-298, 1943.
51. _____, and Bodman, G.B., "Field Measurements of Water Movement Through a Silt Loam Soil", J. Am. Soc. Agron. 33: 713-731, 1941.
52. Fair, G.M., and Hatch, L.P., "Fundamental Factors Governing the Streamline Flow of Water Through Sand", J. Am. Water Wks. Assoc. 25: 1551-1565, 1933.
53. Gardner, W., "The Movement of Moisture in Soils by Capillarity", Soil Sci., 7: 313-319, 1919.
54. _____, "A Capillary Transmission Constant and Methods of Determining it Experimentally", Soil Sci., 10: 103-127, 1920.

55. _____ , "The Capillary Potential and Its Relation to Soil Moisture Constants", Soil Sci. 10: 357-360, 1920.
56. _____ , and Widstoe, J.A., "The Movement of Soil Moisture", Soil Sci. 11: 215-232, 1921.
57. _____ , "The Role of Capillary Potential in the Dynamics of Soil Moisture", J. Agri. Res. 53: 57-61, 1936.
58. Gardner, W.H., and Hsieh, J.C., "Water Velocity in Unsaturated Porous Materials", Soil Sci. Soc. Amer. Proc. 20: 157-161, 1956.
59. _____ , and Gardner, W., "Flow of Soil Moisture in the Unsaturated State", Soil Sci. Soc. Amer. Proc. 15: 42-50, 1951.
60. Gardner, W.R., "Calculation of Capillary Conductivity from Pressure Plate Outflow Data", Soil Sci. Soc. Amer. Proc. 20: 317-320, 1956.
61. _____ , "Some Steady-State Solutions of the Unsaturated Moisture Flow Equation with Application to Evaporation from a Water Table", Soil Sci. 85: 228-232, 1958.
62. _____ , "Solutions of the Flow Equation for the Drying of Soils and Other Porous Media", Soil Sci. Soc. Amer. Proc. 23: 183-187, 1959.
63. _____ , and Fireman, M., "Laboratory Studies of Evaporation from Soil Columns in the Presence of a Water Table", Soil Sci. 85: 244-249, 1958.

64. Geuze, E.C.W.A., deBruyn, C.M.A., and Joustra, K.,
"Results of Laboratory Investigations on the Electrical
Treatment of Soils", Proc. II Int. Conf. Soil Mech.
Found Engg. 3: 153-157, 1948.
65. Green, L., and Dewez, P., "Fluid Flow Through Porous
Metals", J. Appl. Mech., March, 1951.
66. Gurr, C.G., Marshall, T.J., and Hutton, J.T., "Movement
of Water in Soil Due to a Temperature Gradient",
Soil Sci. 74: 335-345, 1952.
67. Habib, P., "Thermo-osmosis", Translated by D.A. Sinclair,
NRC (Canada), Tech. Translation 708, 1957.
68. Happel, J., and Byrne, B.J., "Steady Motion of a Sphere
in a Cylindrical Tube", Ind. Eng. Chem. Vol. 46,
no. 6: 1181-1187, June, 1954.
69. Highway Research Board, "A Co-operative Study of
Structural Design of Non-rigid Pavements", SR 46,
Washington 1959.
70. Hutcheon, W.L., "Characteristics of Water Movement
in Unsaturated Soils", Proc. Conf. Building Res.,
NRC (Canada), DRB Bull. 1, 1955.
71. _____, "Moisture Flow Induced by Thermal Gradients
Within Unsaturated Soils", Highway Res. Board,
SR 40: 113-133, 1958.
72. Hutchison, H.P., Nixon, I.S., and Denbeigh, K.G.,
"The Thermo-osmosis of Liquids Through Porous Materials",
Disc. Faraday Soc. 3: 86-93, 1948.

73. Jones, H.E., and Kohnke, H., "The Influence of Soil Moisture Tension on Vapour Movement of Soil Water", Soil Sci. Soc. Amer. Proc. 16: 245-248, 1952.
74. Kanaku, P.E., "The Effect of Initial Moisture in a Soil on Moisture Movement", Soil Sci. 10: 143-153, 1920.
75. Keen, B.A., "The System Soil - Soil Moisture", Trans. Faraday Soc. 17 (2): 228-243, 1922.
76. _____, "The Physical Properties of Soils", New York: Longmans Public. Co., 1931.
77. Kirkham, D., and Feng, C.L., "Some Tests on the Diffusion Theory and Laws of Capillary Flow in Soils", Soil Sci. 67: 27-40, 1949.
78. Klute, A., "A Numerical Method for Solving the Flow Equation for Water in Unsaturated Materials", Soil Sci. 73: 105-116, 1952.
79. _____, "Some Theoretical Aspects of Flow of Water in Unsaturated Soils", Soil Sci. Soc. Amer. Proc. 16: 144-148, 1952.
80. _____, Bruce, R.R., and Russel, M.B., "The Application of the Diffusivity Concept to Soil Moisture Movement", Trans. VI Int. Cong. Soil Sci. 1: 345-352, 1956.
81. _____, and Wilkinson, G.E., "Some Tests of the Similar Media Concept of Capillary Flow...", Soil Sci. Soc. Amer. Proc. 22: 278-281, 1958.

82. Kresten, M.S., "Structural Design of Non-rigid Pavements - Survey of Subgrade Moisture", Proc. Highway Res. Board 24: 497-511, 1944.
83. _____, "Progress Report of Special Project on Structural Design of Non-rigid Pavements - Subgrade Moisture Conditions Beneath Airport Pavements", Proc. Highway Res. Board 25: 450-461, 1945.
84. Kuzmak, J.M., and Sereda, P.J., "On the Mechanism by which Water Moves Through a Porous Material Subjected to a Temperature Gradient", Highway Res. Board, SR 40: 134-146, 1958.
85. Lebedeff, A.F., "The Movement of Ground and Soil Waters", I Int. Cong. Soil Sci. 1: 459-494, 1927.
86. Low, P.F., "Force Fields and Chemical Equilibrium in Heterogeneous Systems with Special Reference to Soils", Soil Sci. 71: 409-418, 1951.
87. _____, "Effect of Osmotic Pressure on Diffusion Rate of Water", Soil Sci. 80, 1955.
88. _____, and Lovel, C.W., "The Factor of Moisture in Frost Action", Highway Res. Board, Bull. 225, 1959.
89. Luthin, J.N., and Miller, R.D., "Pressure Distribution in Soil Columns Draining into Atmosphere", Soil Sci. Soc. Amer. Proc. 17: 1953.
90. Marwick, A.H.D., and Dobson, A.F., "Application of Electro-osmosis to Soil Drainage", Road Abstr. 12 (207) 1945.

91. Miller, R.D., and Miller, E.E., "A Physical Theory of Capillary Flow Phenomena", J. Appl. Phys. 27: 324-332, 1956.
92. _____, and _____, "Theory of Capillary Flow: I Practical Implications", Soil Sci. Soc. Amer. Proc. 19: 267-271, 1955.
93. _____, and _____, "Theory of Capillary Flow: II Experimental Information", Soil Sci. Soc. Amer. Proc. 19: 271-275, 1955.
94. Murayama, S., and Mise, T., "On the Electro-chemical Consolidation of Soils Using Aluminum Electrodes", Proc. III Int. Conf. Soil Mech. Found. Engg. 1: 157-159, 1953.
95. Muskat, M., "The Flow of Homogeneous Fluids Through Porous Media", New York: McGraw-Hill, 1937.
96. Nutting, P.G., "Physical Analysis of Oil Sands", Bull. Amer. Assoc. Petrol. Geol. 14: 1337-, 1930.
97. Penman, H.L., "Gas and Vapour Movements in the Soil - Diffusion of Vapours Through Porous Solids", J. Agri. Sci. 30:437-462, 1940.
98. Philip, J.R., "The Concept of Diffusion Applied to Soil Water", Proc. National Acad. Sciences (India), Vol. 24 (A I), 1955.
99. _____, "Numerical Solution of Equation of the Diffusion Type of Diffusivity Concentration-Dependent", Trans. Faraday, Soc. 51: 885-892, 1955.

100. _____, and deVries, D.A., "Moisture Movement in Porous Materials under Temp. Gradients", Trans. Am. Geophys. Union, 38 (2): 222-232, 1957.
101. _____, "Physics of Water Movement in Porous Solids", Highway Res. Board, SR 40: 147-163, 1958.
102. Piaskowski, A., "Investigations on Electro-osmotic Flow in Soils in Relation to Different Characteristics", Proc. IV Int. Conf. Soil Mech. Found. Engg. I: 89-93, 1957.
103. Porter, A.W., and Rao, P.A.M., "The Law of Capillary Flow in the Case of Colloids", Trans. Faraday Soc. 23: 311-314, 1927.
104. Purcell, W.R., "Capillary Pressures - Their Measurement Using Mercury and the Calculation of Permeability Therefrom", Trans. Amer. Inst. Min. Metall. Engrs. 186: 39-48, 1949.
105. Preece, E.F., "Geotechnics and Geotechnical Research", Proc. Highway Res. Board 27: 384-417, 1947.
106. Quirk, J.P., and Schofield, R.K., "The Effect of Electrolyte Concentration on Soil Permeability", J. Soil Sci. 6: 163-178, 1955.
107. Rollins, R.A., "The Development of Non-homogeneous Flow Conditions During Electro-osmosis in a Clay Mineral", Proc. Highway Res. Board, 35: 686-692, 1956.
108. _____, Spangler, M.G., and Kirkham, D., "Movement of Soil Moisture Under a Thermal Gradient", Proc. Highway Res. Board, 33: 492-508, 1954.
109. Richards, L.A., "Concerning Permeability Units for Soils", Soil Sci. Soc. Amer. Proc. 5: 49-53, 1939.

110. _____ , "Methods of Measuring Soil Moisture Tension",
Soil Sci . 68: 95-112, 1949.
111. Richards, S.J., and Weeks, L.V., "Capillary Conductivity
Values from Moisture Yield and Tension Measurements on
Soil Columns", Soil Sci . Soc. Amer. Proc. 17: 206-9, 1951.
112. Russel, M.B., and Klute, A., "Movement of Water in Soils",
Agric. Engg. 35 (11): 808-810, 1954.
113. _____ , and Spangler, M.G., "The Energy Concept
of Soil Moisture and Mechanics of Unsaturated Flow",
Proc. Highway Res. Board, 21: 435-449, 1941.
114. Scheidegger, A., "The Physics of Flow Through Porous
Media", Toronto: University of Toronto Press, 1957.
115. Schofield, R.K., "The p_F of the Water in Soil", Trans.
III Int. Cong. Soil Sci . 2: 37, 1935.
116. _____ , and da Costa, J.V.B., "Measurement of the
 p_F in Soil by Freezing Point", J. Agri. Scie. 28: 644-652,
1938.
117. Schmid, W.E., "The Permeability of Soils and the Concept
of a Stationary Boundary Layer", Proc. A.S.T.M. p. 1206,
1957.
118. _____ , "Water Movement in Soils Under Pressure
Potentials", Highway Res. Board, SR 40: 164-177, 1958.
119. Shukla, K.P., "Electrochemical Treatment of Clays",
Proc. III Int. Conf. Soil Mech. Found Engg. 1: 199-202,
1953.
120. Shull, C.A., "Osmotic Phenomena", Trans. Faraday Soc.
17 (2): 255-259, 1922.

121. Smith, W.O., "Thermal Conductivities in Moist Soils",
Soil Sci . Soc. Amer. Proc. 4: 32-40, 1939.
122. Spangler, M.G., "Some Problems in Subgrade Moisture
Control", Proc. Highways Res. Board 25: 434-450, 1945.
123. Spiel, S., and Thompson, M.R., "Electrophoretic Dewatering
of Clay", Trans. Electrochem. Soc. 81: 119-145, 1942.
124. Staple, W.J., and Lehane, J.J., "Movement of Moisture
in Unsaturated Soils", Canad. J. Agri. Sci .
34: 329-342, 1954.
125. Swenson, E.G., and Sereda, P.J., "Preliminary Experiments
on the Movement of Water...due to a Temperature Gradient",
Proc. Conf. Building Res., NRC (Canada) DBR Bull.
1: 102-110, 1955.
126. Swertzendruber, D., "Anamolies in Capillary Intake as
Explained by Capillary Rise Experiments", Soil Sci . Soc.
Amer. Proc. 20: 453-458, 1956.
127. _____, Deboodt, M.F., and Kirkham, D., "Capillary
Intake Rate of Water and Soil Structure", Soil Sci . Soc.
Amer. Proc. 18, 1954.
128. _____, and Kirkham, D., "Capillary Fringe and Flow
of Water in Soils", Soil Sci . 81: 473-484, 1956.
129. _____, and _____, "Capillary Fringe and Flow
of Water in Soil: II Experimental Results", Soil Sci .
82: 91-95, 1956.
130. Taylor, D.W., "Fundamentals of Soil Mechanics", New York:
John Wiley, 1958.

131. Taylor, S.A., and Cavazza, L., "The Movement of Soil Moisture in Reponse to a Temperature Gradient", Soil Sci . Soc. Amer. Proc. 18: 351-357, 1954.
132. Terzaghi, K.V., and Peck, R.B., "Soil Mechanics in Engineering Practice", New York: John Wiley, 1958 (Art. 4, 11 and 39) .
133. Thompson, M.R., "Principles of Microelectrophoresis Cell", Trans. Electrochem. Soc. 81: 147-164, 1942.
134. Vallentine, H.R., "Applied Hydrodynamics", London: Butterworths Scientific Publications, 1959 (Chap. 2) .
135. Winterkorn, H.F., "Mechanism of Water Attack on Dry Cohesive Soil Systems", Soil Sci. 54, 1942.
136. _____ , "Fundamental Similarities Between Electro-osmotic and Thermo-osmotic Phenomena", Proc. Highway Res. Board, 27: 443-455, 1947.
137. _____ , "Potentials in Moisture Migration", Proc. Conf. Building Res., NRC (Canada) DBR Bull. 1: 86-101, 1955 (a) .
138. _____ , "Water Movement Through Porous Hydrophilic Systems Under Capillary, Electric and Thermal Potentials" A.S.T.M. STP 163: 27-35, 1955 (b) .
139. _____ , "Mass Transport Phenomena in Moist Porous Systems as Viewed from the Thermodynamics of Irreversible Processes", Highway Res. Board, SR 40: 324-338, 1958.

140. _____ , and Eyring, H., "Theoretical Aspects of Water Accumulation in Cohesive Subgrade Soils",
Proc. Highway Res. Board, 25: 422-433, 1945.
141. Woodside, W., and Kuzmak, J.M., "Effect of Temperature Distribution on Moisture Flow in Porous Materials",
Trans. Am. Geophys. Union, Vol. 39 (4): 676-680, 1958.
142. Woodward, G.O., and Miller, W., "A Study of Soil Water Movement by Electro-osmosis", Agric. Eng., Jan., 1953.

APPENDIX A

HYDRODYNAMICS OF SOIL MOISTURE

A.1. Basic equations of classical hydrodynamics:

Every system involving flow of fluids must obey three basic equations; the equation of continuity, the equation of state, and the Newtonian equation of motion. The equation of continuity is the same as the well-known principle of conservation of matter, restated in algebraic terms. The equation of state characterises the fluid under consideration and the thermodynamic nature of flow: if fluid is a liquid, the liquid may be incompressible or compressible; if the fluid is a gas, the flow may be either adiabatic or isothermal. An algebraic statement of the particular nature of the flow and of the fluid constitutes the equation of state. The equation of motion specifies the nature of interaction between the fluid and the flow region or medium.

The law of conservation of matter states that the fluid mass in any closed system can be neither created nor destroyed. For analytical purposes, this may be restated as follows: 'The net excess of mass flux, per unit of time, into or out of any infinitesimal volume element in the fluid system is exactly equal to the change per unit of

time of the fluid density in that element, multiplied by the free volume of that element' (Muskat 1937). This statement takes the form of the equation

$$\operatorname{div}(\rho \bar{v}) = \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = -n \frac{\partial \rho}{\partial t} \quad (1)$$

where \bar{v} is the velocity vector at any point with components v_x , v_y , v_z , in the three co-ordinate directions, ρ is the fluid density, and n is the porosity of the medium. If the motion is steady with respect to time, the right side of equation (1) vanishes and the equation reduces to

$$\operatorname{div}(\bar{v}) = 0 \quad (2)$$

The equation of state will, in general, be a quantitative statement of the relationship between the density ρ , the pressure p , and the absolute temperature T of the fluid, in the form of an equation:

$$\Phi(\rho, p, T) = 0 \quad (3)$$

For example, if one is dealing with strictly incompressible liquids, the statement that the density is constant is an equation of state. In the case of ideal gases, the gas law, $p v = RT$ is an equation of state.

In addition to the equation characterising the physical nature of the fluid, we must also have an equation which will characterise the thermodynamic nature of the flow system, so that one of the variables may be eliminated. For homogeneous fluids in viscous flow, Muskat gives the following equation:¹

$$\rho = \rho_0 \left(\frac{p}{p_0} \right)^m \cdot e^{\beta_f (p - p_0)} \quad (4)$$

¹. The equation given by Muskat is of a slightly different form; the form used here is taken from Scheidegger's monograph (1957).

where ρ_0 is the density of the fluid under a pressure p_0

ρ is the density under a pressure p

m & β_f are constants.

The particular set of conditions may be signified as follows:

Liquids: $m = 0$

Incompressible liquids. $\beta_f = 0$

Compressible liquids. $\beta_f \neq 0$

Gases: $m \neq 0$, $\beta_f = 0$

Isothermal process $m = 1$

Adiabatic process $m = c_v / c_p$

where c_v and c_p are the specific heats of the gas under constant volume and constant pressure respectively.

For incompressible liquids, equation (4) reduces to and substitution in the equation of continuity yields

$$\text{div} (\bar{v}) = 0 \quad (5)$$

which now involves the velocity components only.

Having formulated the fundamental equation of continuity and of state, it now remains to characterise the flow dynamically, and state explicitly how the fluid reacts to pressure gradients and external forces. This involves the statement of the Newtonian law of motion in hydrodynamical terms. Newton's law states that the force acting on any body is equal to the mass of the body times the acceleration of the body in the direction of the force. Considering the case of flow of

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description of the object of the study.

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fluids, the external forces acting on the mass of fluid of unit volume may be separated into three parts: the pressure gradients having components in the three co-ordinate directions; the 'body forces' due to gravity and other causes, having components F_x , F_y , F_z ; the viscous retarding forces due to internal resistance experienced by the fluid.

In text-books on hydrodynamics, it has been shown that the final equations relating all these forces to the mass and acceleration of the body may be written as

$$\begin{aligned}\rho \frac{Dv_x}{Dt} &= \rho F_x - \frac{\partial p}{\partial x} + \frac{1}{3} \mu \frac{\partial \theta}{\partial x} + \mu \nabla^2 v_x, \\ \rho \frac{Dv_y}{Dt} &= \rho F_y - \frac{\partial p}{\partial y} + \frac{1}{3} \mu \frac{\partial \theta}{\partial y} + \mu \nabla^2 v_y, \text{ and,} \\ \rho \frac{Dv_z}{Dt} &= \rho F_z - \frac{\partial p}{\partial z} + \frac{1}{3} \mu \frac{\partial \theta}{\partial z} + \mu \nabla^2 v_z\end{aligned}\quad (6)$$

where, in addition to the symbols defined previously,

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \frac{dx}{dt} \frac{\partial}{\partial x} + \frac{dy}{dt} \frac{\partial}{\partial y} + \frac{dz}{dt} \frac{\partial}{\partial z} \equiv \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z},$$

$$\theta \equiv \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z},$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

and μ is the co-efficient of viscosity of the fluid.

Equations (6) are known as the Navier-Stokers' equations, which are the hydrodynamical equivalent of the Newtonian law of motion. (For a rigorous derivation of these equations, see Lamb 1879, Vallentine 1959). The terms $\frac{Dv_x}{Dt}$, $\frac{Dv_y}{Dt}$, and $\frac{Dv_z}{Dt}$ represent the acceleration of the unit volume of the fluid in the three co-ordinate directions; the velocity of the fluid element changes during any interval of time at the position it occupied originally; and, during this interval, the

element itself has moved to a new position, which involves an additional change in velocity, thus giving rise to the total time derivative of the velocity. The first two terms on the right side of the equations need no explanation. The third term is an expression for the viscous resistance arising from the fact that the three velocity components have gradients in all the three directions at any instant; the fourth term is also a part of the expression for the viscous resistance, and has to be included because the gradients of the velocity components of any element depends on time as well as the position of the element in space.

Thus, we have five equations - one equation of continuity, one equation of state and three equations of motion - and five unknown variables - p , ρ , v_x , v_y , v_z . (The variable, temperature, can be eliminated between the equation of state and the thermodynamic equation characterising the flow.) These five equations should be, and indeed are, sufficient to solve for the five unknowns. However, the mathematical difficulties, arising from the fact that the Navier-Stokes equations are not linear, exclude the possibility of obtaining a general solution for all but the simplest of problems with relatively simple geometry of the flow region (Muskat 1937, Vallentine 1959). Thus, for a straight capillary tube of circular cross-section, the Navier-Stokes equations can be solved exactly, yielding the well-known Poiseuille's law:

$$Q = \frac{\pi}{8} \frac{\Delta p}{h} \frac{a^4}{\mu} \quad (7)$$

where, 'Q' is the volume rate of flow through the capillary,

' Δp ' is the pressure drop across the capillary,

'h' is the length and 'a' is the radius of the capillary.

A few approximate solutions of the Navier-Stokes equations have been obtained for specific problems: Thus, Happel and Byrne (1954) obtained an expression for the velocity field produced by the presence of a sphere at the axis of a circular tube through which the fluid is flowing. The expression was based on the classical equations of hydrodynamics, but the inertia term $-\frac{D\bar{v}}{Dt}$ was dropped out. Using this expression, the authors derived a simple equation for the pressure drop across dilute systems of spherical assemblages, assuming the force exerted on each sphere to be the same as the force that would be exerted if the sphere were located at the axis of the tube. Their final equation is of the form:

$$\Delta p_s = 9(1-e) \cdot \frac{\eta \mu L}{a^2} \quad , \quad (8)$$

or, transposing,

$$v = \frac{a^2 \Delta p_s}{9 \mu (1-e) L} \quad (9)$$

In these equations, Δp_s is the pressure drop across the assemblage of spheres,

v is the macroscopic velocity in the tube approaching the assemblages of spheres,

e is the fractional void volume,

L is the length of the assemblage in the direction of flow,

and a is the radius of the spheres.

It may be noticed that in the case of incompressible liquids, the average velocity through the tube must be the same as the average macroscopic velocity through the assemblage, from considerations of the continuity of flow.

Unfortunately, no experimental evidence has been adduced by the authors to support the exactness of their final equation, and, as such, the validity of their assumptions is not known. The qualification of the spherical assemblage - as dilute systems - is not clearly defined. Presumably, this refers to the case where the total volume occupied by the spheres is negligible in comparison with the volume of the fluid involved in the flow; if this were so, the usefulness of this analysis is severely restricted as far as practical problems are concerned, since the conditions obtaining in the field are, in most cases, just the reverse of what has apparently been assumed in the analysis. And, finally, the analysis is only for the case of a system of spheres of equal radius, and the limited extent of applicability of the analysis to soil-water systems is quite obvious.

From the foregoing, it will be obvious that an analysis of the problem from the standpoint of classical hydrodynamics is beset with many mathematical difficulties the circumvention of which involves a large number of questionable assumptions. Hence, it is not surprising that attempts should have been made to solve the problem from a semi-empirical approach. In this semi-empirical approach,

the first two equations of classical hydrodynamics - namely, the equation of continuity and the equation of state - have to be retained, since these two equations represent the fundamental laws that must be obeyed by all systems regardless of the nature of the approach. So, the difference between the classical and the semi-empirical approaches should lie in the formulation of the equation of motion. While the classical approach adopts the Navier-Stokes equation, the semi-empirical approach adopts the experimental equation given by Darcy as the starting point.*

A.2. Semi-empirical approach based on Darcy's law:

Since Darcy's law was obtained from experiments with unidirectional flow of water through filter beds, a generalized version of the law, as applicable to three dimensional flow systems involving any type of fluid has to be formulated. For this purpose, the fundamental equation of Darcy is taken in its alternative form, namely,

$$v = \frac{K}{\mu} \cdot \frac{dp}{dx} \quad (10)$$

where, besides other quantities as already defined, K is the so-called co-efficient of permeability with the units of (distance).² In this form, the co-efficient of permeability is a property of the porous medium only, and is independent of the properties of the percolating fluid.

(cf. Chap. II of this dissertation.)

* The original authorship of this semi-empirical approach based on Darcy's law is not clearly known; the analysis to be presented in the following pages is largely based on the work of Muskat (cf. Muskat 1937).

As a first step, it is assumed that 'the resultant velocity at any point may be resolved into three components, parallel to the three co-ordinate axes, each reacting to the pressure gradients independently of the others' (Muskat 1937). Then Darcy's law may be rewritten as follows:

$$\begin{aligned}v_x &= K_x / \mu \cdot \frac{\partial p}{\partial x} \\v_y &= K_y / \mu \cdot \frac{\partial p}{\partial y} \\v_z &= K_z / \mu \cdot \frac{\partial p}{\partial z}\end{aligned}\tag{11}$$

Equations (11) imply that the medium is strictly anisotropic. However, solutions of practical problems are always obtained on the assumption that the medium is isotropic and the final answers are modified suitably to account for the anisotropy of the medium.* Hence, the medium will be treated as isotropic in the following analysis, on the understanding that a transformed geometry of the porous medium is to be used in the case of anisotropic porous media.

It is evident that when the velocity of flow has a component in the vertical direction, the effect of the acceleration due to gravity will have to be taken into account. In addition, if there are adsorptive force fields present, their influence on the flow should also be taken into account.

*As an example, the conventional method of sketching flownets may be cited. When the medium through which flow is taking place is anisotropic, the procedure involves the modification of the scale of the diagram so as to obtain a transformed section that is isotropic, sketching the flow net on this transformed section, and then transforming the flow net back to the original scale.

These forces are collectively termed as body forces,* and, in the present case, they include all forces except the ones due to fluid pressure difference. If we assume that the body forces have components F_x , F_y , and F_z in the three co-ordinate directions, equations (11) may be modified as:

$$\begin{aligned} v_x &= K / \mu \cdot \left(\frac{\partial p}{\partial x} - F_x \right) \\ v_y &= K / \mu \cdot \left(\frac{\partial p}{\partial y} - F_y \right) \\ v_z &= K / \mu \cdot \left(\frac{\partial p}{\partial z} - F_z \right) \end{aligned} \quad (12)$$

If, further, we assume that the body forces can be represented by a potential V , such that

$$\bar{F} = F_x + F_y + F_z = - \left(\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} \right) \quad (13)$$

equations (12) may be written as

$$\begin{aligned} v_x &= K / \mu \cdot \frac{\partial}{\partial x} (p + V) \\ v_y &= K / \mu \cdot \frac{\partial}{\partial y} (p + V) \\ v_z &= K / \mu \cdot \frac{\partial}{\partial z} (p + V) \end{aligned} \quad (14)$$

$$\text{Setting } \phi = K / \mu \cdot (p + V), \quad (15)$$

equations (14) become

* It may be mentioned in passing that this grouping of so many forces into one single factor makes it very difficult to take explicit account of the individual components of the body forces and of the effects physical changes on those components.

$$v_x = \frac{\partial \phi}{\partial x}$$

$$v_y = \frac{\partial \phi}{\partial y} \quad (16)$$

$$v_z = \frac{\partial \phi}{\partial z}$$

$$\text{or} \quad \bar{v} = \nabla \phi \quad (17)$$

Muskat calls ' ϕ ' as the velocity potential, since the gradient of ϕ in any one of the co-ordinate directions gives the velocity of the fluid in that direction. Equations (16), or its vector equivalent (equation (17)) is the final form of generalised Darcy's law and may be considered as the macroscopic equivalent for the Navier-Stokes equations of classical hydrodynamics. (Muskat 1937)

Now that the dynamical basis for the analysis of problems involving viscous flow of fluids through porous media have been formulated in the form of eq. (16), it is pertinent to examine the assumptions that have been made in the analysis. Basically, it is assumed that Darcy's law is valid for the case under consideration. In an earlier section of this report, the limitations of the law have been enumerated (cf. Chap. II). The generalised form of the law is subject to the same limitations. It has been assumed that the three component velocities of any element of fluid react to the respective pressure gradients in the three co-ordinate directions in exactly the same way as postulated in the fundamental Darcy's law. The use of a velocity potential function presupposes that the body forces can be

expressed in the form of equation (13). In the case of gravity, the vector of the gravitational force field is always directed vertically downwards and its magnitude is solely governed by the mass of the element considered and its position in the force field with respect to some arbitrarily chosen datum. The magnitudes and directions of the vectors of the remaining components of the body forces - such as those due to chemical and electrical potentials - are dependent upon the specific conditions of the system and cannot be uniquely defined once and for all.

After the dynamical equation of motion has been formulated (Eq. 17), it is only necessary to combine it with the equations of continuity and of state to obtain the final differential equation describing the flow. The continuity equation is

$$\text{div.} (\rho \bar{v}) = -n \frac{\partial \rho}{\partial t} \quad (\text{Eq. 1})$$

and the generalised form of the equation of state is

$$\rho = \rho_0 \left(\frac{p}{p_0} \right)^m \cdot e^{\beta_f (p - p_0)} \quad (\text{Eq. 4})$$

For steady flow conditions, the right side of equation (1) vanishes; for liquids, $m = 0$ in equation (4), and if the liquid is incompressible, $\beta_f = 0$, thus reducing equation (4) to

$$\rho = \rho_0 \quad (18)$$

Substituting this in Eq. (1), we obtain

$$\text{div} (\bar{v}) = 0 \quad (19)$$

and, substituting for \bar{v} from equation (17),

$$\text{div} (\nabla \phi) = 0$$

$$\text{or,} \quad \nabla^2 \phi = 0 \quad (20)$$

In equation (20), ∇^2 is the Laplacian operator, given by $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. Similar expressions can be set up for the flow of compressible liquids and gases. In the case of incompressible liquids, the time-dependent quantities disappear, and as such, no non-steady states within the system can be present, unless the boundary conditions themselves vary with time. In contrast, the equations describing the flow of compressible liquids and gases involve the time-derivative and so their solutions are much more complicated. However, the time derivatives do vanish under steady flow conditions in this case also, and the final equations are similar to the Laplacian equation (20), the difference being only in the form of the dependent variable. For practical purposes, it will always suffice to treat steady state liquid-flow systems as problems involving flow of incompressible liquids alone, in view of the very low compressibilities of normal liquids (Muskat 1937).

A.3. Methods of solution:

The solution of equation (20) can be obtained in a number of ways: If the geometry of the flow region is simple enough to be expressed analytically, a purely analytical solution is possible,

employing the principles of conformal mapping, Green's functions or the method of successive relaxation (for details, see Muskat 1937, Klute 1952, Scheidegger 1957, Schmid 1958). Such solutions require a very high degree of mathematical skill, and, in view of the uncertainties involved in the description of the porous medium, it is doubtful whether such mathematical precision is warranted or not.

A simpler method of solution is suggested by the fact that Eq. (20) is similar to many equations describing physical processes such as flow of heat and of electricity; solutions depending on the analogy between the flow of water and that of heat or of electricity are called analog solutions; these methods involve the experimental determination of such physical quantities as magnetic flux, electric current, electric potential, or temperature, which, by their analog interpretation, give the velocity and force potential of the seepage flow problem. On account of the high degree of accuracy obtainable, electric analogs are widely employed in preference to others.

A third method of solution, suggested by Forcheimer, involves the solution of the Laplacian equation by a graphical procedure of trial and error (see A. Casagrande 1937; also Taylor 1958). This method is based on the fact that the solution of the Laplacian equation yields two families of orthogonal curves; the 'flow net' is sketched with due attention to the boundary and initial conditions of the problem and to the requirement that each 'field' of the flow net must approximate

a square. This method gives reliable results in a mere fraction of the time required for analytical solutions, but it cannot be used for solving three-dimensional flow problems, unless there is radial symmetry (Schmid 1958).

Finally, solutions may be obtained from mechanical models of the flow problem; however, this method is very expensive, and on account of the experimental errors involved, the results obtained therefrom are not fully reliable.

Flow systems involving steady state flow of liquids with a free boundary - as, for example, seepage through dams - are very difficult to solve analytically. Analog solutions are particularly useful in such cases. Graphical solutions by the Forcheimer method requires that the location of the free boundary be guessed in the first place; a few empirical methods are available for this purpose (see Casagrande 1937, Taylor 1958). Problems involving flow of gases through porous media are outside the scope of this disseration; for information on these problems, the reader is referred to the books of Muskat, Scheidegger, and Carman (1956).

APPENDIX B

CONCERNING DARCY'S COEFFICIENT OF PERMEABILITY

B.1. Experimental determination of the saturated permeability:

B.1.1. Various methods and optimum range of application;

Owing to the importance of Darcy's law in most of the problems involving flow of fluids through porous media, it is only natural that several methods should have been devised for the determination of the Darcy's coefficient of permeability of the medium. There are, at present, six different methods for this determination in the case of soils, and each particular method gives good results over a particular range of permeability values. The various methods and their generally accepted optimum ranges of application are listed below:

Method	Range in terms of permeability in cm/sec.
a. Constant Head Permeability test	10^2 to 10^{-3}
b. Variable Head Permeability test	10^{-3} to 10^{-5}
c. Capillary Permeability test	10^{-1} to 10^{-5}
d. Estimation from grain size analysis data	10^2 to 10^{-3}
e. Computation from consolidation test data	10^{-7} to 10^{-9}
f. Field tests including pumping test and ground velocity tests	10^2 to 10^{-3}

B.1.2 Critical evaluation of the merits of the various tests:

The constant head permeability test is probably the simplest of all tests and the test arrangement is very similar to the one employed by Darcy in his experiments. The conditions of steady-state percolation are definitely established and thus there can be no question of the applicability of Darcy's law for the evaluation of the data from the test. Some errors are likely to result from the formation of filter skins of fine material at the entrance and exit ends of the specimen. In one type of apparatus, this has been eliminated by providing pressure tappings within the specimen and beyond the influence of the filter skins; the hydraulic head causing flow is then taken as the pressure difference between two such tappings, active over that portion of the specimen between the two tappings. With relatively impermeable soils, loss of outflow water due to evaporation may introduce considerable errors.

The variable head permeability test is by far the most versatile of all tests, giving good results over a large range of permeability values. But the head causing flow through the soil is varying continuously with time, and thus the flow is not under steady-state conditions. The use of Darcy's law (which is valid for steady-state percolation only) in the evaluation of the test data may seem to be inherently erroneous. However, it should be pointed out that, in the analysis, an infinitesimal change in the water level in the stand pipe is related to the flow through the soil in an infinitesimal interval of time (as given by Darcy's law) in order to set up a differential equation; during this

infinitesimal interval of time, one may assume that steady state flow conditions are prevalent. The values obtained from a solution of such a differential equation may not be reliable if the permeability varies with the pressure; such variations with pressure have been observed in the case of flow of gases (Scheidegger 1957), but the writer is not aware of any recorded evidence of significant variations in the case of flow of liquids. Nonetheless, it is desirable to keep the total change of pressure head during any one experiment as small as possible.

Both constant head and variable head permeability tests can be run with either undisturbed or remolded specimens. In either case, it is essential to have complete saturation of the specimen since the permeability has been found to be very sensitive to changes in the degree of saturation (Klute, 1952). In undisturbed specimens, it is seldom possible to have full saturation for the following reasons:

1. Due to natural resistance to wetting, some air is entrapped on the surface of the grains during the formation of the deposit;
2. If the soil is organic, the pore water may contain dissolved gases which will come out of solution when the overburden pressure is released.

With remolded samples, a high degree of saturation is usually attainable by first evacuating the dry soil and then saturating it with distilled de-aired water.

The capillary permeability test can be conducted on remolded specimens only since the soil must be in a dry state at the start of the

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experiment. Since the pores of the specimen cannot be evacuated prior to the test (one end of the specimen must be in free communication with the atmosphere), some air is always entrapped in the voids during the test; consequently, complete saturation of the specimen by capillarity is never realized in the test and the value of the permeability obtained from such tests should be interpreted as valid for the particular degree of saturation obtained in the test. In the actual test, the rate of advance of the wetting front in the specimen is observed under two different pressure conditions in order to obtain two simultaneous equations involving the two unknown quantities --- permeability and another (fictitious) quantity called 'effective capillary head'; this necessitates an abrupt change in the pressure head causing flow. Depending upon the magnitude of the change in the pressure head between the two stages, there will be changes in the volume of the entrapped air, thus giving rise to corresponding changes in the degree of saturation. For this reason, considerable skill is required in setting up the test conditions in order to obtain reliable results.

Estimation of the value of the permeability of a soil from its grain size or similar properties is not an accurate method, but the accuracy so obtained may be sufficient for many practical cases. This method is suitable for coarse grained soils only, that do not possess a marked degree of "structure".

Estimation of the permeability from the consolidation character-

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istics of the soil is primarily based on the validity of the Terzaghi theory of consolidation. It is well known that the theory is based on many simplifying assumptions, and "while the theory is extremely successful in determining the magnitude of settlements, predicted and observed settlement rates - where the permeability comes into play - often differ considerably" - (Schmid, 1957). Thus the indirect determination of the permeability from consolidation test data can be regarded as semi-quantitative only. Here again, except in problems related with fundamental research on permeability, the degree of accuracy obtained from this method is sufficient for almost all practical cases.

Field tests for permeability include the well-known pumping test and the ground water velocity tests. Both types of tests are rather expensive and as such are limited for use in major projects only. In fine-grained soils, a long time may be required for the attainment of steady state conditions. However the results obtained from such tests are far more reliable than the other laboratory methods; moreover, the field tests yield the average permeability of a large mass of soil deposit and "the results are representative of the average properties of the soil in its natural state". (Taylor, 1958).

B.2 Various theories of permeability:

On account of the importance of the permeability in so many fields of science, numerous attempts have been made to correlate the permeability with other physical properties of the porous medium.

Such attempts (at correlation of the permeability with the porosity or the pore structure or the grain size distribution or similar properties of the porous medium) have never been successful (Scheidegger 1957), and the empirical correlations claimed to have been 'found' are, at best, valid over a very narrow range of values only.

Having failed to obtain a unique correlation between permeability and some other physical property of the porous medium, many investigators attempted to idealize the pore space of a porous medium and describe the permeability as a function of some physical parameter of the idealized model. The advantage of this approach is that, by choosing the proper type of model, the problem can be treated mathematically. One set of such models consist of a bundle of straight capillary tubes, arranged in various ways, and the mathematical treatment stems from the well-know Poiseuille's law of flow through straight circular capillary tubes. This idealization is hardly justifiable, since the pores of a natural porous medium do not even remotely resemble straight circular capillary tubes.

Another type of models are based on the assumption that the pores are analogous to a series of a channels. It will be recalled that the modified coefficient of permeability has the dimensions of a length squared (Eq.2.6). This length, characteristic of the porous medium, is taken as the equivalent of the 'hydraulic radius' of the porous medium. A possible measure of the hydraulic radius of a porous

medium will be the ratio of the volume to the surface area of the pore space. Slitcher (1899) considered a bed of spherical particles as equivalent to the porous medium and arrived at the equation

$$K = \frac{N^2 d^2}{96(1-n)} \quad (1)$$

where, N = the ratio of the void area to the total area in a plane section, when the plane passes through the centres of adjacent spheres,

d = the diameter of the spheres, and

n = the porosity.

Slitcher's work has been subsequently refined and extended by Graton and Fraser (1935) and others, but the general approach may be criticised on the grounds that for a natural porous medium, a unique value for the 'diameter of the sphere' cannot be assigned; further in a bed of spherical particles of equal diameter, there can be no interconnected voids and consequently no flow through the pores (Scheidegger, 1957).

Kozeny replaced the porous medium by an assemblage of channels of various cross-sections but of a definite length, solved the Navier-Stoke's equations simultaneously for all channels passing through a cross-section normal to the general direction of flow, and expressed the permeability as a function of the specific surface of the porous medium (Scheidegger, 1957). The final expression obtained by Kozeny is of the form

$$K = c \cdot n \cdot s^{-3/2} \quad (2)$$

where, c = a dimensionless constant, called the 'Kozeny constant',

n = the porosity of the medium, and

s = the specific surface of the porous medium.

The Kozeny equation has been modified by a large number of people, foremost among them being Carman (1937), who set:

$$K = n^3 / 5 s_0^2 (1-n^2) \quad (3)$$

where s_0 is the 'surface exposed to the liquid per unit volume of the solid (and not the porous) material. By implication, the value of the Kozeny constant is $1/5$. Equation (9), called the Kozeny-Carman equation has been extensively used for the determination of the surface area of industrial powders (Carman 1939).

Yet another approach to a physical characterization of the permeability may be called the drag theory of permeability (Scheidegger 1957). Here, the walls of the porous medium are assumed to be obstacles to an otherwise straight flow of a viscous fluid. Using Navier - Stoke's equations, the drag on each portion of the wall is estimated and the total drag is set equal to the resistivity (μ/K , as per Darcy's law) of the porous medium to flow of fluids. This approach has been found to give reliable results in highly porous media such as textile fibres, where each fibre can be regarded as solitary within the fluid. (Scheidegger, loc. cit.).

The problem has been tackled from the standpoint of statistical

mechanics, assuming the porous medium to be intrinsically disordered. Childs and George (1948) were probably the first to imply such an approach and Scheidegger (1957) has given a comprehensive outline of the method. For the details of this approach, Scheidegger's monograph may be consulted.

Recently, Schmid (1957) has applied the concept of a stationary boundary layer and of ineffective porosity to the problem of percolation of water through soils and has shown that there is a linear relationship between permeability and porosity. His final expression is of the form:

$$k = (\gamma / 32 u) \cdot D_E^2 (n - n_0) \quad (4)$$

where, k = the Darcy's coefficient of permeability in velocity units,

n = the total porosity

n_0 = the effective porosity, and the other symbols are as already defined.

The symbol ' D_E ' is defined as the 'effective diameter of the pore' and the method of determining the same is not described. It may be mentioned that the experimental results reported by the author seem to agree well with the theoretical considerations, in that the plot of permeability against porosity turns out to be linear except when the total porosity is approximately equal to the effective porosity, where 'additional phenomena occur which need further clarification'.

Thus, there are so many theories, purportedly aimed at ascribing a physical significance to the permeability of porous media.

The writer fails to detect any practical usefulness of such theories except the indirect determination of the surface area of industrial powders. It is decidedly easier to determine the permeability by direct experiments than to measure the so-called physical parameters - tortuosity factor, effective diameter of grains, effective pore size and so on - and use them in complex formulae of doubtful reliability. It would seem that the time spent in formulating such vague theories may well be profitably utilized in perfecting the technique of experimental determination of the permeability and of the associated problems of sampling and sample preparation.

B.3 Factors influencing the permeability of soils:

As already mentioned, the nature of the definition of the permeability as adopted in civil engineering practice renders the same as dependent upon the properties of the soil as well as those of the percolating liquid. The two most important properties of the percolating liquid that are of interest, are the unit weight and viscosity of the liquid. In the case of water - which is the only liquid of interest for a civil engineer - the unit weight may be regarded as constant for most practical purposes, and so, variations of its viscosity alone need be considered. By far the most important physical agency that influences the viscosity is the temperature of the liquid, the viscosity decreasing with increasing temperatures. This suggests a relation of the form:

$$k_1 : k_2 = \mu_2 : \mu_1 \quad (5)$$

which has been found to be valid for coarse-grained soils. In clays, temperature seems to have a greater influence on the viscosity of the pore water than in the case of coarser soils (Terzaghi and Peck, 1958 ed., pp. 47). When the interstitial distance between particles become smaller and smaller, the viscosity of the water in these interstices increases from that of the free water at the same temperature; Terzaghi gives the following empirical equation for the increase in viscosity in narrow slits:

$$\mu / \mu_0 = 1 + \frac{6.02 \times 10^{-42}}{S^8} \quad \text{to} \quad 1 + \frac{2.42 \times 10^{-43}}{S^8} \quad (6)$$

where, μ_0 is the viscosity of water at 25 C., and

μ is the viscosity of water in a slit of width $2S$ at the same temperature (reported by Bull and Moyer, 1936).

Thus, in fine grained clay soils and soil colloids, Eq. (11) cannot be assumed to hold.

As regards the properties of the porous medium that influence the permeability, it is obvious that the geometry of the pore space should be the foremost factor to be considered. Unfortunately, there is no unique definition of the 'pore geometry' of a porous medium. Such a definition should include: the size, the shape, the number and orientation of the particles, the amount of sealed pores, and the shape of the continuous voids. Then it is apparent that the permeability should be a function of each one of these variables. It may be recalled that many attempts at empirical correlation between permeability and some other

physical property of the porous medium have proved to be unsuccessful; the various physical properties - porosity, grain size, void ratio, 'average pore size', 'average grain size' and so on - are but individual manifestations of a composite property, namely, the pore geometry; the failure of the above-mentioned attempts may be due to the fact that only one of the physical properties was considered as a variable in any one problem without paying due regard to the fact that more than one variable are required to characterize the pore geometry. Such a complete characterization of the pore geometry appears impossible since the constituent properties are interrelated in a complex and as yet unknown manner.

For a very comprehensive discussion of the factors influencing the permeability of soils, the reader is referred to Chap. VI of Taylor's text book on Fundamentals of Soil Mechanics (1958).

APPENDIX C

THERMODYNAMICS APPLIED TO SOIL WATER SYSTEMS

C.1. Fundamental laws:

Classical thermodynamics is based on two fundamental laws governing transformation and conservation of energy. The first law of thermodynamics - the law of conservation of energy - may be stated quantitatively as follows: The increase in internal energy of a substance during any transformation equals the amount of heat taken in, less the work done by the substance; or algebraically,

$$de = dq - dw \quad (1)$$

Thus, the first law specifies quantitatively, the relation that must exist during any interchange of work and heat.

While mechanical work can always be transformed into heat, the reverse process of transforming heat into mechanical work is subject to further restrictions; these restrictions are laid down by the second law of thermodynamics: "No self-acting device, unaided by an external agency, can transfer heat from a body of lower temperature to one of higher temperature" (Edlefson and Anderson, 1943). Alternatively, the law may be restated as follows: Work can never be produced in a reversible cycle of changes operating in surroundings, all at

the same temperature. The former statement specifies the direction in which heat will flow to do work, while the latter statement specifies the condition under which no work can be done.

C.2. Definitions of term used:

C.2.1. Reversible operations:

A thermodynamically reversible process is one that is carried out so slowly, that the system is in continuous equilibrium with its surroundings. The process can be made to proceed in the opposite direction by an infinitesimal change in anyone of the conditions governing the state of the substance and the direction of the process - such as temperature or pressure. Since the forces responsible for the process are of infinitesimal magnitude, the process never reaches equilibrium. All natural processes are irreversible to a larger or smaller extent, and thus the thermodynamically reversible process is only an ideal case.

C.2.2. Entropy:

The quantity 'entropy' is defined as the ratio of the amount of heat taken in by a substance, to the temperature (in the Kelvin Scale) at which it is taken in; in symbolic form,

$$ds = \frac{dq}{T} \quad \text{or} \quad \Delta s = \frac{\Delta q}{T} \quad (2)$$

By convention, 'the amount of heat' is taken as positive when heat flows into the substance, and negative when it flows out of the substance into the surroundings. It can be shown that the total change in entropy of a substance in any reversible cycle of operations is always zero and that

the same is always greater than zero in any irreversible cycle of changes; the entropy of a substance undergoing irreversible processes always tends to a maximum positive value (see Edlefsen and Anderson, 1943 for a rigorous proof of these statements).

C.2.3. Free energy:

The "free energy" of a substance is defined as the difference between the total energy of the substance and the amount of unavailable energy; in other words, it is the amount of energy readily available for doing useful work. Stated algebraically,

$$f = e + Pv - Ts \quad (3)$$

where, f is the free energy of the substance,

e is the internal energy of the substance,

P is the external pressure acting on the substance

v is the specific volume of the substance,

T is the absolute temperature, and

s is the change in entropy of the system.

In a perfectly reversible process, the change in entropy is zero, and the free energy becomes identical with the total energy of the substance; as the degree of irreversibility increases, entropy also increases, and the free energy decreases. It is to be noted that an irreversible process is characterized by a negative change in free energy or a positive change in entropy.

From the second law of thermodynamics - which specifies the

to the fact that the system is not in equilibrium with the environment. The system is in a state of non-equilibrium, and the process of relaxation to equilibrium is a slow one. The system is in a state of non-equilibrium, and the process of relaxation to equilibrium is a slow one. The system is in a state of non-equilibrium, and the process of relaxation to equilibrium is a slow one.

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$$(1) \quad \frac{dN}{dt} = -\lambda N$$

where N is the number of particles, and λ is the decay constant.

The solution of the equation is given by

$$N = N_0 e^{-\lambda t}$$

where N_0 is the initial number of particles, and t is the time.

The half-life of the system is given by

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

where $t_{1/2}$ is the half-life, and λ is the decay constant.

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direction in which work may be performed - and from the observed fact that free water tends to move into a partially saturated soil spontaneously, we may conclude that the free energy of water in a partially saturated soil is less than that of pure water. By convention, the free energy of pure water outside the zone of influence of any adsorptive force field (excepting that due to gravity) is taken as zero, and this forms the datum of free energy for soil moisture studies; then, the free energy of soil-water in a partially saturated soil is always negative; it reaches a maximum value of zero when the soil is completely saturated. This negative quantity has been variously called soil suction, pressure deficiency, soil water tension, capillary potential and so on; by common usage, the negative sign is usually omitted. In the following sections, the term 'free energy' will be used consistent with the practice in thermodynamic analyses.

C.3. Characteristics of free energy:

C.3.1. Free energy of several phases in equilibrium:

In the preceding paragraph, it was seen that the free energy of soil water is less than that of pure, free water and that free water tends to move into a partially saturated soil spontaneously. Generalizing, we may state that any system containing two or more substances in co-existence, cannot remain in equilibrium unless the free energies of the individual substances are equal to each other. Thus, in a partially saturated soil in a state of thermodynamic equilibrium, the free energy

of the soil water is equal to that of the water vapour in contact with it; and, in a partially frozen soil, the free energy of the unfrozen water is equal to that of the ice formed as well as that of the water vapour in contact with it. This offers a convenient way of determining the free energy of soil water by measuring the pressure of water vapour in equilibrium with it.

C.3.2. Relation between vapour pressure and free energy:

By definition,

$$f = e + P.v - T.s. \quad (3)$$

In differential form,

$$df = de + Pdv + v dP - T ds - s dT \quad (4)$$

According to the first law of thermodynamics,

$$de = dq - dw$$

Separating the total work done, dw , into two component parts - a work of expansion against a constant pressure, P , equal to $P.dv$, and a mechanical work done, dw_m , Eq. (1) becomes

$$de = dq - P.dv - dw_m \quad (5)$$

Further, by definition,

$$ds = \frac{dq}{T} \quad \text{or, } dq = T ds \quad (\text{Eq. 2})$$

Substituting these into equation (4) and simplifying,

$$dg = v dP - s dT - dw_m \quad (6)$$

If the temperature remains constant, and if no mechanical work is performed in the process, Eq. (6) reduces to

$$df = v dP \quad (7)$$

the same result as in the case of the first two cases. In the case of the third case, the result is different. In this case, the result is that the function $f(x)$ is not continuous at $x=0$. This is because the function $f(x)$ is not defined at $x=0$. The function $f(x)$ is defined for all $x \neq 0$, but it is not defined at $x=0$. Therefore, the function $f(x)$ is not continuous at $x=0$.

Let us now consider the case where $f(x)$ is defined for all x . In this case, the function $f(x)$ is continuous at $x=0$. This is because the function $f(x)$ is defined at $x=0$, and the limit of $f(x)$ as x approaches 0 is equal to $f(0)$.

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$$x_0 = 0$$

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Assuming that the behaviour of water vapour can be approximated by the ideal gas equation,

$$P.v = RT \quad (8)$$

Eq. (7) may be rewritten as

$$df = RT \frac{dP}{P} \quad (9)$$

Integrating between two phases A and B,

$$f_A - f_B = RT \ln \frac{P_A}{P_B} \quad (10)$$

Letting the subscript B refer to the datum, f_B equals zero, and P_B equals the saturated water vapour pressure, designated by P_o . Then,

$$f_A = RT \ln \frac{P_A}{P_o} \quad (11)$$

where in f_A is the free energy of the soil water vapour,

P_A is the pressure of soil water vapour, and

R is the universal gas constant.

C.3.3 Effect of temperature on free energy:

From Eq. (6), assuming that no mechanical work is performed, we obtain, for conditions of constant pressure,

$$df = -s dT \quad (12)$$

or,

$$\left(\frac{\partial f}{\partial T} \right)_P = -s \quad (13)$$

Thus, the rate of change of free energy of a substance, due to change in temperature, equals the negative of the entropy of the substance.

C.3.4. Effect of hydrostatic pressure on free energy:

This effect is readily seen by rewriting Eq. (7) as

$$\frac{\partial f}{\partial P} = v \quad (14)$$

Thus, the rate of change of free energy due to changes in pressure equals the specific volume of the substance. In particular, if we have pure water and ice in co-existence, the following numerical values apply:

$$\frac{\partial f}{\partial P} = 1.000, \text{ for water, and}$$

$$\frac{\partial f}{\partial P} = 1.090, \text{ for ice.}$$

An increase in pressure causes a greater increase in the free energy of ice than in that of water on account of the different specific volumes.

As the free energy tends to be a minimum, the mass of ice will spontaneously melt if the pressure is increased, since, by doing so the rate of decrease in free energy is kept smaller. On the other hand, the rate of decrease of free energy of ice, with decreasing pressure is greater than that of water; hence, the mass of water will freeze spontaneously, if the pressure is reduced, again for the same reason as above.

C.3.5. Effect of dissolved salts on free energy of a solution:

Rewriting Eq. (7) in terms of infinitesimal quantities,

$$\Delta f = f_B - f_A = \int_{P_A}^{P_B} v \, dP \quad (15)$$

Here phase A refers to the solution while phase B refers to the pure solvent and the above equation gives the free energy of the solution with respect to that of the pure solvent. In our particular case, the 'pure solvent' is the stress-free, pure water and the 'solution' is the soil-water containing dissolved salts. If the external pressure acting on both the pure solvent and the solution is the same, P_B may be set equal to zero, while P_A equals the osmotic pressure, (ΔP_o), due to the presence of dissolved salts. Assuming the specific volume of the solvent to be constant (the compressibilities of normal liquids are very low), equation

(15) reduces to

$$\begin{aligned}\Delta f &= f_B - f_A = v \int_{\Delta P_o}^0 dP \\ &= -v (\Delta P_o)\end{aligned}\quad (16)$$

Then,

$$\frac{\partial f}{\partial (\Delta P_o)} = -v \quad (17)$$

or, the rate of change of free energy of a solvent due to changes in the osmotic pressure of the solution, equals the negative of the specific volume of the solvent. Since the specific volume is always a positive quantity, it is seen that the free energy always decreases with increasing osmotic pressure, that is, with increasing quantity of dissolved materials. In contrast, an increase in hydrostatic pressure increases the free energy of the substance.

C.4. Freezing point depression of soil water:

C.4.1. Preliminary considerations:

It has been found (Bouyoucos 1917) that "some of the water in the

soil refused to freeze, and the amount varied with the class of soil, degree of supercooling, etc." Subsequent investigations by Bouyoncos and others have revealed that the freezing point of water in soils is lower than that of pure water under the same conditions of pressure and that the extent of depression is related to the energy state of the soil water. Nowadays, the freezing point depression (FPD) is accepted as one of the common means of estimating the free energy of soil moisture (see Schofield 1935, Bodman and Day 1938, Schofield and da Costa 1938). The thermodynamic interrelationships between the free energy and the FPD have been lucidly presented by Edlefson and Anderson (1943). In the following sections, a brief outline of these relationships are given; for detailed information, the reader is referred to the paper of Edlefson and Anderson.

C.4.2. Relation between FPD and hydrostatic pressure:

We will consider the case of a pure liquid remaining in thermodynamic equilibrium with its solid phase under atmospheric pressure conditions; the temperature of the system will then be freezing point of the liquid and the free energies of the two phases will be equal to each other. Due to an increase in the external pressure on the system, the equilibrium is disturbed, and a new state of equilibrium can be set up, only if the temperature of the system is suitably altered. Then the final free energies of the two phases will also be equal, since they are again in equilibrium with each other. Thus, the starting point is the

statement that the initial free energies and the changes in the free energies of the two phases are separately equal, that is,

$$f_1 = f_2 \text{ and } df_1 = df_2$$

Substituting for df from equation (6), and stipulating that no mechanical work is done in the process,

$$v_1 dp - s_1 dT = v_2 dp - s_2 dT \quad (18)$$

Simplifying,

$$\frac{dP}{dT} = \frac{s_2 - s_1}{v_2 - v_1} = \frac{\Delta s}{\Delta v} \quad (19)$$

By definition,

$$\Delta s = \frac{\Delta q}{T}$$

From the first law of thermodynamics,

$$de = dq - P \cdot dv - dw_m \quad (\text{Eq. 5})$$

and, since no mechanical work is performed, this may be rewritten as

$$dq = de + P \cdot dv \quad (20)$$

$$= d(e + Pv) = dh \quad (21)$$

In equation (21), the quantity, $h = e + P \cdot v$, is defined as the 'Heat content' of the substance. If we are dealing with changes of state of anyone substance, the heat content will become the commonly known 'latent heat of fusion' or the 'latent heat of condensation' of the substance.

From Eq. (21) and the definition of entropy,

$$s_2 - s_1 = \Delta s = \frac{\Delta h}{T} \quad (22)$$

Substituting in Eq. (19),

$$\frac{dP}{dT} = \frac{\Delta h}{\Delta v \cdot T} \quad (23)$$

or,

$$\frac{\Delta P}{\Delta T} = \frac{\Delta h}{\Delta v \cdot T} \quad (24)$$

Thus, the relation between any change in external (hydrostatic) pressure and the corresponding change in the equilibrium temperature of a system is given by Eq. (23); in this equation,

Δh is the change in the heat content of the substance,

Δv is the change in the specific volume of the substance,

T is the absolute equilibrium temperature (initial),

ΔP is the change in the external pressure, and

ΔT is the change in the equilibrium temperature.

As a particular case, when water freezes to form ice, its volume increases, and so Δv is positive; but an amount of heat equal to the latent heat of fusion of ice is given out in the process, and so Δh is negative. Thus, Eq. (23) predicts that the equilibrium temperature of an ice-water mixture is lowered by increasing the hydrostatic pressure on the system.

Combining equation (23) with equation (7), we obtain,

$$f = \frac{v \cdot \Delta h}{\Delta v \cdot T} \cdot \Delta T \quad (24)$$

which gives the change in free energy of a substance due to a change in the hydrostatic pressure, as measured by the FPD.

C.4.3. Relation between FPD and osmotic pressure;

Here again, we start with the same initial and final conditions, that is, $f_1 = f_2$; and $df_1 = df_2$, wherein, f_1 is the initial free energy of a pure solvent in thermodynamic equilibrium with its solid phase, whose free energy is f_2 . The addition of some soluble salts to the solvent decreases its free energy, as given by Eq. (17). Let us designate the increase in osmotic pressure of the solvent due to the presence of dissolved salts by ΔP_o .

The specific free energy of the solid phase depends only on the temperature, since the external pressure conditions are assumed to remain unaltered; consequently, we may write,

$$df_2 = \left(\frac{\partial f_2}{\partial T} \right)_P dT \quad (25)$$

and, in view of Eq. (13),

$$df_2 = -s_2 dT \quad (26)$$

The specific free energy of the solution, on the other hand, depends both upon the temperature and the osmotic pressure; hence,

$$df_1 = \left(\frac{\partial f_1}{\partial T} \right)_{P_o} dT + \left(\frac{\partial f_1}{\partial P_o} \right)_T dP_o \quad (27)$$

Substituting from Eq. (13) and (17),

$$df_1 = -s_1 dT - v_1 dP_o \quad (28)$$

Since

$$df_1 = df_2, \text{ we also have:}$$

$$-s_2 dT = -s_1 dT - v_1 dP_o$$

$$\text{or,} \quad \frac{d P_o}{dT} = - \frac{(s_1 - s_2)}{v} \quad (29)$$

$$= - \frac{dh}{v \cdot T} \quad (30)$$

$$\text{Hence,} \quad dP_o = - \frac{dh}{v \cdot T} \cdot dT$$

$$\text{or,} \quad \Delta P_o = - \frac{\Delta h}{v \cdot T} \cdot \Delta T \quad (31)$$

$$\text{Since} \quad f = - v \cdot \Delta P_o \quad \text{Eq. (17),}$$

$$\text{we have} \quad f = \frac{\Delta h}{T} \cdot \Delta T \quad (32)$$

which expresses the change in free energy of a solution with respect to that of the pure solvent, in terms of the FPD caused by the presence of dissolved salts.

C.5. Methods of determining free energy of soil water:

Several methods have been evolved for the determination of the so-called capillary potential of soil moisture, and all these methods may be readily used for the evaluation of the free energy of the soil moisture. For a brief description of the principles involved in the various methods, the reader is referred to Chap. II of this dissertation; a comprehensive evaluation of the merits and demerits of these various methods has been recently published (Croney et. al. 1952).

C.6. Movement of soil moisture:

In the preceding sections, some of the characteristics of the

thermodynamic function "free energy" and its interrelationships with other physical quantities such as temperature, hydrostatic pressure and osmotic pressure were outlined. In this closing section, the manner in which these relationships may be utilized to predict movement of water through soils will be outlined.

Under isothermal conditions, if two sections of moist soil in contact with each other do not have the same free energy, moisture will tend to migrate spontaneously, so as to equalize the energies. The direction of migration is specified by the second law of thermodynamics: it will be in the direction of decreasing free energy. An incidental point is that the final free energy of the system as a whole will have decreased by a certain amount since the change is spontaneous and not perfectly reversible.

In analogy with the "potential concept" of driving forces, the negative of the gradient of the total specific free energy of the soil moisture is the force per unit mass of water causing movement. Incorporating this in a Darcy type equation,

$$v = k \nabla (\Delta f_s) \quad (33)$$

where v is the steady state velocity of flow, and

k is the proportionality factor, the permeability.

Combining this equation with the continuity requirement, an equation relating the moisture content of the soil and the gradient for free energy may be set up; the solution of this equation for particular boundary and initial

conditions of the problem will give the mode of distribution of water within the flow region.

There is one difficulty yet to be overcome in this analysis: the final equation involves moisture density, time, the rate of change of free energy, and the permeability, all of which are variables. In order to solve the problem completely, four independent equations are required, or else it is necessary to eliminate two of the variables by some other means. Many investigators have assumed some single-valued relationships between moisture density on the one hand, and the permeability and the capillary potential (analogous to free energy) on the other hand (see Chap. II). On account of the lack of perfect reversibility, these assumptions are not strictly justified. Under these circumstances, there is a great need for expressing thermodynamically irreversible functional relationships in terms of easily measurable physical parameter; once such parameters have established, we would have taken a big step forward in the solution of this complex problem.

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